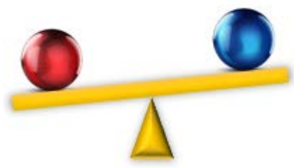
# L.4 Linear Inequalities and Interval Notation



Mathematical inequalities are often used in everyday life situations. We observe speed limits on highways, minimum payments on credit card bills, maximum smartphone data usage per month, the amount of time we need to get from home to school, etc. When we think about these situations, we often refer to limits, such as "a speed limit of 100 kilometers per hour" or "a limit of 1 GB of data per month." However, we don't have to travel at exactly 100 kilometers per hour on the highway, or use exactly 1 GB of data per month. The limit only establishes a boundary for what is allowable. For example, a driver traveling $x$ kilometers per hour is obeying the speed limit of 100 kilometers per hour if $x \leq 100$ and breaking the speed limit if $x > 100$. A speed of $x = 100$ represents the boundary between obeying the speed limit and breaking it. Solving linear inequalities is closely related to solving linear equations because equality is the boundary between *greater than* and *less than*. In this section we discuss techniques needed to solve linear inequalities and ways of presenting these solutions.

## Linear Inequalities

**Definition 4.1** ► A **linear inequality** is an inequality with only **constant** or **linear terms**.
A linear inequality in one variable can be written in one of the following forms:

$$\boldsymbol{Ax + B > 0}, \quad \boldsymbol{Ax + B \geq 0}, \quad \boldsymbol{Ax + B < 0}, \quad \boldsymbol{Ax + B \leq 0}, \quad \boldsymbol{Ax + B \neq 0},$$

for some real numbers $A$ and $B$, and a variable $x$.

A variable value that makes an inequality true is called a **solution** of this inequality. We say that such variable value **satisfies** the inequality.

**Example 1** ► **Determining if a Given Number is a Solution of an Inequality**

Determine whether each of the given values is a solution of the inequality.

**a.** $3x - 7 > -2$; 2, 1

**b.** $\frac{y}{2} - 6 \geq -3$; 8, 6

**Solution** ►

**a.** To check if 2 is a solution of $3x - 7 > -2$, replace $x$ by 2 and determine whether the resulting inequality $3 \cdot 2 - 7 > -2$ is a true statement. Since $6 - 7 = -1$ is indeed larger than $-2$, then 2 satisfies the inequality. So 2 is a solution of $3x - 7 > -2$.

After replacing $x$ by 1, we obtain $3 \cdot 1 - 7 > -2$, which simplifies to the false statement $-4 > -2$. This shows that 1 in not a solution of the given inequality.

**b.** To check if 8 is a solution of $\frac{y}{2} - 6 \geq -3$, substitute $y = 8$. The inequality becomes $\frac{8}{2} - 6 \geq -3$, which simplifies to $-2 \geq -3$. Since this is a true statement, 8 is a solution of the given inequality.

Similarly, after substituting $y = 6$, we obtain a true statement $\frac{6}{2} - 6 \geq -3$, as the left side of this inequality equals to $-3$. This shows that $-3$ is also a solution of the original inequality.

Usually, an inequality has an infinite number of solutions. For example, one can check that the inequality

$$2x - 10 < 0$$

is satisfied by $-5$, 0, 1, 3, 4, 4.99, and generally by any number that is smaller than 5. So in the above example, the set of all solutions, called the **solution set**, is infinite. Generally, the solution set to a linear inequality in one variable can be stated either using **set-builder notation**, or **interval notation**. Particularly, the solution set of the above inequality could be stated as $\{x|x > 5\}$, or as $(-\infty, 5)$.

In addition, it is often beneficial to visualize solution sets of inequalities in one variable as graphs on a number line. The solution set to the above example would look like this:


For more information about presenting solution sets of inequalities in the form of a graph or interval notation, refer to *Example 3* and the subsection on "*Interval Notation*" in *Section R2* of the *Review* chapter.

To solve an inequality means to find all the variable values that satisfy the inequality, which in turn means to find its solution set. Similarly as in the case of equations, we find these solutions by producing a sequence of simpler and simpler inequalities preserving the solution set, which eventually result in an inequality of one of the following forms:

$$x > constant, \quad x \geq constant, \quad x < constant, \quad x \leq constant, \quad x \neq constant.$$

**Definition 4.2** ► **Equivalent inequalities** are inequalities with the same solution set.

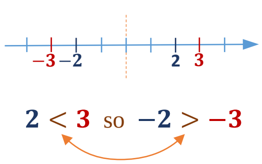
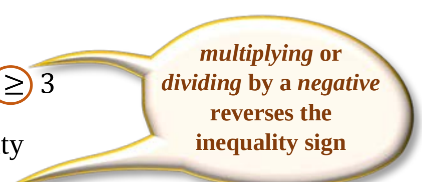



**Figure 1**

Generally, we create equivalent inequalities in the same way as we create equivalent equations, **except for *multiplying* or *dividing* an inequality by a *negative* number**. Then, we ***reverse the inequality symbol***, as illustrated in *Figure 1*.

So, if we multiply (or divide) the inequality

$$-x \geq 3$$




by $-1$, then we obtain an equivalent inequality

$$x \leq -3.$$

We encourage the reader to confirm that the solution set to both of the above inequalities is $(-\infty, -3]$.

Multiplying or dividing an inequality by a positive number, leaves the inequality sign unchanged.

The table below summarizes the basic inequality operations that can be performed to produce equivalent inequalities, starting with $A < B$, where $A$ and $B$ are any algebraic expressions. Suppose $C$ is a real number or another algebraic expression. Then, we have:

| Inequality operation | General Rule | Example |
|---|---|---|
| **Simplification** | Write each expression in a simpler but equivalent form | $2(x-3)<1+3$ can be written as $2x-6<4$ |
| **Addition** | if $A<B$ then $A+C<B+C$ | if $2x-6<4$ then $2x-6+6<4+6$ |
| **Subtraction** | if $A<B$ then $A-C<B-C$ | if $2x<x+4$ $2x-x<x+4-x$ |
| **Multiplication** **when multiplying by a negative value, reverse the inequality sign** | if $C>0$ and $A<B$ then $CA<CB$ if $C<0$ and $A<B$ then $CA>CB$ | if $2x<10$ then $\frac{1}{2}\cdot 2x<\frac{1}{2}\cdot 10$ if $-x<-5$ then $x>5$ |
| **Division** **when dividing by a negative value, reverse the inequality sign** | if $C>0$ and $A<B$ then $\frac{A}{C}<\frac{B}{C}$ if $C<0$ and $A<B$ then $\frac{A}{C}>\frac{B}{C}$ | if $2x<10$ then $\frac{2x}{2}<\frac{10}{2}$ if $-2x<10$ then $x>-5$ |

**Example 2** ► **Using Inequality Operations to Solve Linear Inequalities in One Variable**

Solve the inequalities. Graph the solution set on a number line and state the answer in interval notation.

**a.** $\frac{3}{4}x+3>15$ **b.** $-2(x+3)>10$

**c.** $\frac{1}{2}x-3\le\frac{1}{4}x+2$ **d.** $-\frac{2}{3}(x-3)-\frac{1}{2}\ge\frac{1}{2}(5-x)$

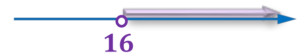
**Solution** ► **a.** To isolate $x$, we apply inverse operations in reverse order. So, first we subtract the 3, and then we multiply the inequality by the reciprocal of the leading coefficient. Thus,

$$\frac{3}{4}x+3>15 \quad / -3$$

$$\frac{3}{4}x>12 \quad / \cdot\frac{4}{3}$$

$$x>\frac{\overset{4}{\cancel{12}}\cdot 4}{\cancel{3}}=16$$

To visualize the solution set of the inequality $x > 16$ on a number line, we graph the interval of all real numbers that are grater than 16.



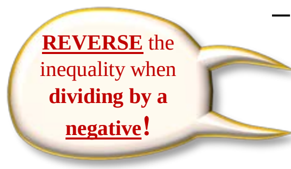

Finally, we give the answer in interval notation by stating $\boldsymbol{x \in (16, \infty)}$. This tells us that any $x$-value greater than 16 satisfies the original inequality.

*Note:* The answer can be stated as $x \in (16, \infty)$, or simply as $(16, \infty)$. Both forms are correct.

**b.** Here, we will first simplify the left-hand side expression by expanding the bracket and then follow the steps as in *Example 2a*. Thus,

$$-2(x+3) > 10$$

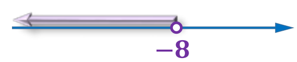
$$-2x - 6 > 10 \qquad / -3$$




$$-2x > 16 \qquad / \div(-2)$$

$$x < -8$$

The corresponding graph looks like this:




The solution set in interval notation is $(-\infty, \boldsymbol{-8})$.

**c.** To solve this inequality, we will collect and combine linear terms on the left-hand side and free terms on the right-hand side of the inequality.

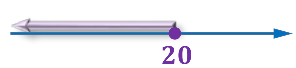
$$\frac{1}{2}x - 3 \le \frac{1}{4}x + 2 \qquad / -\frac{1}{4}x, +3$$

$$\frac{1}{2}x - \frac{1}{4}x \le 5$$

$$\frac{1}{4}x \le 5 \qquad / \cdot 4$$

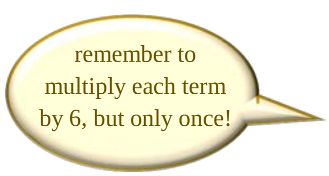
$$x \le 20$$

This can be graphed as




and stated in interval notation as $(-\infty, \boldsymbol{20}]$.

**d.** To solve this inequality, it would be beneficial to clear the fractions first. So, we will multiply the inequality by the LCD of 3 and 2, which is 6.




$$-\frac{2}{3}(x-3) - \frac{1}{2} \le \frac{1}{2}(5-x) \qquad / \cdot 6$$

$$-\frac{2 \cdot \cancel{6}^{2}}{\cancel{3}}(x-3) - \frac{1 \cdot \cancel{6}^{3}}{\cancel{2}} \le \frac{1 \cdot \cancel{6}^{3}}{\cancel{2}}(5-x)$$

$$-4(x-3) - 3 \le 3(5-x)$$

$$-4x + 12 - 3 \le 15 - 3x$$

$$-4x + 9 \le 15 - 3x \qquad / \; +4x, -15$$

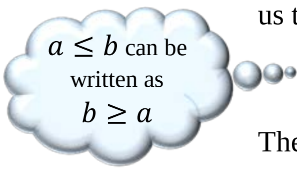
At this point, we could collect linear terms on the left or on the right-hand side of the inequality. Since it is easier to work with a positive coefficient by the $x$-term, let us move the linear terms to the right-hand side this time.

So, we obtain

$$-6 \le 15x \qquad / \; \div 15$$

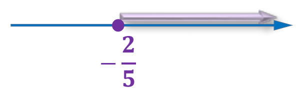
$$\frac{-6}{15} \le x,$$

which after simplifying to $-\frac{2}{5} \le x$ and writing the inequality from right to left, gives us the final result




$$x \ge -\frac{2}{5}$$

The solution set can be graphed as




which means that all real numbers $x \in \left[-\frac{2}{3}, \infty\right)$ satisfy the original inequality.

**Example 3** ► **Solving Special Cases of Linear Inequalities**

Solve each inequality.

**a.** $-2(x-3) > 5 - 2x$ **b.** $-12 + 2(3+4x) < 3(x-6) + 5x$

**Solution** ► **a.** Solving the inequality

$$-2(x-3) > 5 - 2x$$

$$-2x + 6 > 5 - 2x \qquad / \; +2x$$

$$6 > 5,$$

leads us to a true statement that does not depend on the variable $x$. This means that any real number $x$ satisfies the inequality. Therefore, the solution set of the original inequality is equal to all real numbers $\mathbb{R}$. This could also be stated in interval notation as $(-\infty, \infty)$.

**b.** Solving the inequality

$$-12 + 2(3+4x) < 3(x-6) + 5x$$

$$-12 + 6 + 8x < 3x - 18 + 5x$$

$$-6 + 8x < 8x - 18 \qquad / \; -8x$$

$$-6 < -18$$

leads us to a false statement that does not depend on the variable $x$. This means that no real number $x$ would satisfy the inequality. Therefore, the solution set of the original inequality is an empty set $\emptyset$. We say that the inequality has **no solution**.

## Three-Part Inequalities

The fact that an unknown quantity $x$ lies between two given quantities $a$ and $b$, where $a < b$, can be recorded with the use of the three-part inequality $a < x < b$. We say that $x$ is enclosed by the values (or oscillates between the values) $a$ and $b$. For example, the systolic high blood pressure $p$ oscilletes between 120 and 140 mm Hg. It is convenient to record this fact using the three-part inequality $120 < p < 140$, rather than saying that p < 140 and at the same time p > 120. The solution set of the three-part inequality $a < x < b$ or $b > x > a$ is a **bounded** interval $(a, b)$ that can be graphed as


The hollow (open) dots indicate that thy endpoints do not belong to the solution set. Such interval is called **open**.

If the inequality symbol includes equation ($\leq$ or $\geq$), the corresponding endpoint of the interval is included in the solution set. On a graph, this is reflected as a solid (closed) dot. For example, the solution set of the three-part inequality $a \leq x < b$ is the interval $[a, b)$, which is graphed as


Such interval is called **half-open** or **half-closed**.

An interval with both endpoints included is referred to as **closed** interval. For example $[a, b]$ is a closed interval and its graph looks like this


Any three-part inequality of the form

$$constant\ \boldsymbol{a} < (\leq)\ one\ variable\ linear\ \boldsymbol{expression} < (\leq)\ constant\ \boldsymbol{b},$$

where $a \leq b$ can be solved similarly as a single inequality, by applying inequality operations to all of the three parts. When solving such inequality, the goal is to isolate the variable in the middle part by moving all constants to the outside parts.

**Example 4** ► **Solving Three-Part Inequalities**

Solve each three-part inequality. Graph the solution set on a number line and state the answer in interval notation.

**a.** $-2 \leq 1 - 3x \leq 3$ **b.** $-3 < \frac{2x-3}{4} \leq 6$

**Solution** ► **a.** To isolate $x$ from the expression $1 - 3x$, first subtract 1 and then divide by $-3$. These operations must be applied to all three parts of the inequality. So, we have

Remember to **reverse** both inequality symbols when **dividing by a negative** number!

$$-2 \le 1 - 3x \le 3 \quad / -1$$

$$-3 \le -3x \le 2 \quad / \div (-3)$$

$$1 \ge x \ge -\frac{2}{3}$$

The result can be graphed as


The inequality is satisfied by all $x \in \left[-\frac{2}{3}, 1\right]$.

**b.** To isolate $x$ from the expression $\frac{2x-3}{4}$, we first multiply by 4, then add 3, and finally divide by 2.

$$-3 < \frac{2x-3}{4} \le 6 \quad / \cdot 4$$

$$-12 < 2x - 3 \le 24 \quad / +3$$

$$-9 < 2x \le 27 \quad / \div 2$$

$$-\frac{9}{2} < x \le \frac{27}{2}$$

The result can be graphed as


The inequality is satisfied by all $x \in \left(-\frac{9}{2}, \frac{27}{2}\right]$.

## Inequalities in Application Problems

Linear inequalities are often used to solve problems in areas such as business, construction, design, science, or linear programming. The solution to a problem involving an inequality is generally an interval of real numbers. We often ask for the range of values that solve the problem.

Below is a list of common words and phrases indicating the use of particular inequality symbols.

| Word expression | Interpretation |
|---|---|
| ***a*** is less (smaller) than ***b*** | $a < b$ |
| ***a*** is less than or equal to ***b*** | $a \le b$ |
| ***a*** is greater (more, bigger) than ***b*** | $a > b$ |
| ***a*** is greater than or equal to ***b*** | $a \ge b$ |
| ***a*** is at least ***b*** | $a \ge b$ |
| ***a*** is at most ***b*** | $a \le b$ |
| ***a*** is no less than ***b*** | $a \ge b$ |
| ***a*** is no more than ***b*** | $a \le b$ |
| ***a*** is exceeds ***b*** | $a > b$ |
| ***a*** is different than ***b*** | $a \ne b$ |
| ***x*** is between ***a*** and ***b*** | $a < x < b$ |
| ***x*** is between ***a*** and ***b*** inclusive | $a \le x \le b$ |

**Example 5** ► **Translating English Words to an Inequality**

Translate the word description into an inequality and then solve it.

**a.** Twice a number, increased by 3 is at most 9.
**b.** Two diminished by five times a number is between $-4$ and 7

**Solution** ► **a.** Twice a number, increased by 3 translates to $2x+3$. Since "at most" corresponds to the symbol "≤", the inequality to solve is

$$2x+3 \leq 9 \qquad / -3$$

$$2x \leq 6 \qquad / \div 2$$

$$x \leq 3$$

So, all $\boldsymbol{x \in (-\infty, 3]}$ satisfy the condition of the problem.

**b.** Two more than five times a number translates to $2-5x$. The phrase "between $-4$ and 7" tells us that the expression $2-5x$ is enclosed by the numbers $-4$ and 7, but not equal to these numbers. So, the inequality to solve is

$$-4 < 2-5x < 7 \qquad / -2$$

$$-6 < -5x < 5 \qquad / \div(-5)$$

$$\frac{6}{5} > x > -1$$

Therefore, the solution set to this problem is the interval of numbers $\boldsymbol{\left(-1, \frac{6}{5}\right)}$.

Remember to list the endpoints of any interval in *increasing order* (from the smaller to the larger number.)

**Example 6** ► **Using a Linear Inequality to Compare Cellphone Charges**

A cellular phone company advertises two pricing plans. The first plan costs $19.95 per month with 20 free minutes and $0.39 per minute thereafter. The second plan costs $23.95 per month with 20 free minutes and $0.30 per minute thereafter. How many minutes can you talk per month for the first plan to cost less than the second?

**Solution** ► Let $n$ represent the number of cellphone minutes used per month. Since 20 is the number of free-minutes for both plans, then $n-20$ represents the number of paid-minutes. Hence, the expression representing the charge according to the first plan is $19.95+0.39(n-20)$ and according to the second plan is $23.95+0.3(n-20)$.

Since we wish for the first plan to be cheaper, than the inequality to solve is

$$19.95+0.39(n-20) < 23.95+0.3(n-20).$$

To work with 'nicer' numbers, such as integers, we may want to eliminate the decimals by multiplying the above inequality by 100 first. Then, after removing the brackets via distribution, we obtain

$$1995 + 39n - 780 < 2395 + 30n - 600 \qquad / \; -30n, -1215$$

$$1215 + 39n < 1795 + 30n \qquad / \; \div 9$$

$$9n < 580$$

$$n < \frac{580}{9} \approx 64.4$$

So, the first plan is cheaper if less than 65 minutes are used during a given month.

**Example 7** ► **Finding the Test Score Range of the Missing Test**

Ken scored 74% on his midterm exam. If he wishes to get a B in this course, the average of his midterm and final exam must be between 80% and 86%, inclusive. What range of scores on his final exam guarantee him a B in this course?

**Solution** ► Let $n$ represent Ken's score on his final exam. Then, the average of his midterm and final exam is represented by the expression

$$\frac{74 + n}{2}.$$

Since this average must be between 80% and 89% inclusive, we need to solve the three-part inequality

$$80 \le \frac{74 + n}{2} \le 86 \qquad / \; \cdot 2$$

$$160 \le 74 + n \le 172 \qquad / \; -74$$

$$86 \le n \le 98$$

To attain a final grade of a B, Ken's score on his final exam should fall between 86% and 98%, inclusive.

## L.4 Exercises

***Vocabulary Check*** *Complete each blank with one of the suggested words, or with the most appropriate term or phrase from the given list:* ***bounded, closed, interval, open, reversed, satisfy, true, unbounded.***

**1.** We say that a number satisfies one variable inequality if, after substituting this value for the variable, the inequality becomes a ________ statement.

**2.** The solution set of an inequality is the set of all numbers that ______________ the inequality.

**3.** When multiplying or dividing an inequality by a negative value, the inequality symbol must be _____________.

**4.** The solution set of a single inequality is an ______________ interval of real numbers.

**5.** The solution set of a three-part inequality is a ______________ interval of real numbers.

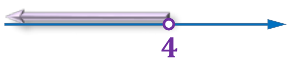
**6.** An ___________ interval does not include its endpoints.

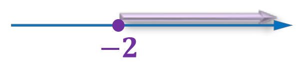
**7.** A ________ interval includes its endpoints.

***Concept Check*** *Using interval notation, record the set of numbers presented on the graph. (Refer to the part "Interval Notation" in section R2 of the Review chapter, if needed.)*

**8.** 


**9.** 


**10.** 


**11.** 


***Concept Check*** *Graph each solution set. For each interval write the corresponding inequality (or inequalities), and for each inequality, write the solution set in interval notation. (Refer to the part "Interval Notation" in section R2 of the Review chapter, if needed.)*

**12.** $(3, \infty)$ **13.** $(-\infty, 2]$ **14.** $[-7,5]$ **15.** $[-1, 4)$

**16.** $x \geq -5$ **17.** $x > 6$ **18.** $x < -2$ **19.** $x \leq 0$

**20.** $-4 < x < 1$ **21.** $3 \leq x < 7$ **22.** $-5 < x \leq -2$ **23.** $0 \leq x \leq 1$

***Concept Check*** *Determine whether the given value is a solution of the inequality.*

**24.** $4n + 15 > 6n + 20; \;\; -5$

**25.** $16 - 5a > 2a + 9; \;\; 1$

**26.** $\frac{x}{4} + 7 \geq 5; \;\; -8$

**27.** $6y - 7 \leq 2 - y; \;\; \frac{2}{3}$

***Concept Check*** *Solve each inequality. Graph the solution set and write the solution using interval notation.*

**28.** $2 - 3x \geq -4$

**29.** $4x - 6 > 12 - 10x$

**30.** $\frac{3}{5}x > 9$

**31.** $-\frac{2}{3}x \leq 12$

**32.** $5(x + 3) - 2(x - 4) \geq 2(x + 7)$

**33.** $5(y + 3) + 9 < 3(y - 2) + 6$

**34.** $2(2x - 4) - 4x \leq 2x + 3$

**35.** $7(4 - x) + 5x > 2(16 - x)$

**36.** $\frac{4}{5}(7x + 6) > 40$

**37.** $\frac{2}{3}(4x - 3) \leq 30$

**38.** $\frac{5}{2}(2a - 3) < \frac{1}{3}(6 - 2a)$

**39.** $\frac{2}{3}(3x - 1) \geq \frac{3}{2}(2x - 3)$

**40.** $\frac{5-2x}{2} \geq \frac{2x+1}{4}$

**41.** $\frac{3x-2}{-2} \geq \frac{x-4}{-5}$

**42.** $0.05 + 0.08x < 0.01x - 0.04(3 - 3x)$

**43.** $-0.2(5x + 2) > 0.4 + 1.5x$

**44.** $-\frac{1}{4}(p + 6) + \frac{3}{2}(2p - 5) \leq 10$

**45.** $\frac{3}{5}(t - 2) - \frac{1}{4}(2t - 7) \leq 3$

**46.** $-6 \le 5x - 7 \le 4$

**47.** $-10 < 3b - 5 < -1$

**48.** $2 \le -3m - 7 \le 4$

**49.** $4 < -9x + 5 < 8$

**50.** $-\frac{1}{2} < \frac{1}{4}x - 3 < \frac{1}{2}$

**51.** $-\frac{2}{3} \le 4 - \frac{1}{4}x \le \frac{2}{3}$

**52.** $-3 \le \frac{7-3x}{2} < 5$

**53.** $-7 < \frac{3-2x}{3} \le -2$

***Concept Check*** *Give, in interval notation, the unknown numbers in each description.*

**54.** The sum of a number and 5 exceeds 12.

**55.** 5 times a number, decreased by 6, is smaller than $-16$.

**56.** 2 more than three times a number is at least 8.

**57.** Triple a number, subtracted from 5, is at most 7.

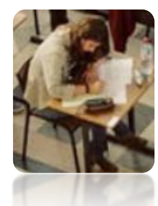
**58.** Half of a number increased by 3 is no more than 12.

**59.** Twice a number increased by 1 is different than 14.

**60.** Double a number is between $-6$ and 8.

**61.** Half a number, decreased by 3, is between 1 and 12.

***Analytic Skills*** *Solve each problem.*



**62.** Amber earned scores of 90 and 82 on her first two tests in Algebra. What score must she make on her third test to keep an average of 84 or greater?

**63.** An average of 70 to 79 in a mathematics class receives a C grade. A student has scores of 59, 91, 85, and 62 on four tests. Find the range of scores on the fifth test that will give the student a C for the course.

**64.** To run an advertisement on a certain website, the website owner charges a setup fee of \$250 and \$12 per day to display the advertisement. If a marketing group has a budget of \$1500 for an advertisement, what is the maximum number of days the advertisement can run on the site?

**65.** A homeowner has a budget of \$150 to paint a room that has 340 $ft^2$ of wall space. Drop cloths, masking tape, and paint brushes cost \$32. If 1 gal of paint will cover 100 $ft^2$ of wall space and the paint is sold only in gallons, what is the maximum cost per gallon of paint that the homeowner can pay?

**66.** The temperature range for a week was between 14°F and 77°F. Using the formula $F = \frac{9}{5}C + 32$, find the temperature range in degrees Celsius.

**67.** The temperature range for a week in a mountain town was between 0°C and 30°C. Using the formula $C = \frac{5}{9}(F - 32)$, find the temperature range in degrees Fahrenheit.

**68.** George earns \$1000 per month plus 5% commission on the amount of sales. George's goal is to earn a minimum of \$3200 per month. What amount of sales will enable George to earn \$3200 or more per month?

**69.** Heritage National Bank offers two different chequing accounts. The first charges \$3 per month and \$0.50 per cheque after the first 10 cheques. The second account charges \$8 per month with unlimited cheque writing. How many cheques can be written per month if the first account is to be less expensive than the second account?

**70.** Glendale Federal Bank offers a chequing account to small businesses. The charge is \$8 per month plus \$0.12 per cheque after the first 100 cheques. A competitor is offering an account for \$5 per month plus \$0.15 per cheque after the first 100 cheques. If a business chooses the first account, how many cheques does the business write monthly if it is assumed that the first account will cost less than the competitor's account?