

Q.2

Applications of Quadratic Equations



Some polynomial, rational or even radical equations are **quadratic in form**. As such, they can be solved using techniques described in the previous section. For instance, the rational equation $\frac{1}{x^2} + \frac{1}{x} - 6 = 0$ is quadratic in form because if we replace $\frac{1}{x}$ with a single variable, say a , then the equation becomes quadratic, $a^2 + a - 6 = 0$. In this section, we explore applications of quadratic equations in solving equations quadratic in form as well as solving formulas containing variables in the second power.

We also revisit application problems that involve solving quadratic equations. Some of the application problems that are typically solved with the use of quadratic or polynomial equations were discussed in *sections F4* and *RT6*. However, in the previous sections, the equations used to solve such problems were all possible to solve by factoring. In this section, we include problems that require the use of methods other than factoring.

Equations Quadratic in Form

Definition 2.1 ▶ A nonquadratic equation is referred to as **quadratic in form** or **reducible to quadratic** if it can be written in the form

$$au^2 + bu + c = 0,$$

where $a \neq 0$ and u represents any *algebraic expression*.

Equations quadratic in form are usually easier to solve by using strategies for solving the related quadratic equation $au^2 + bu + c = 0$ for the expression u , and then solve for the original variable, as shown in the example below.

Example 1 ▶ Solving Equations Quadratic in Form

Solve each equation.

a. $(x^2 - 1)^2 - (x^2 - 1) = 2$ b. $x - 3\sqrt{x} = 10$

c. $\frac{1}{(a+2)^2} + \frac{1}{a+2} - 6 = 0$

Solution ▶ a. First, observe that the expression $x^2 - 1$ appears in the given equation in the first and second power. So, it may be useful to replace $x^2 - 1$ with a new variable, for example u . After this substitution, the equation becomes quadratic,

$$u^2 - u = 2, \quad / -2$$

and can be solved via factoring

$$u^2 - u - 2 = 0$$

$$(u - 2)(u + 1) = 0$$

$$u = 2 \text{ or } u = -1$$

This can be any letter, as long as it is different than the original variable.

Since we need to solve the original equation for x , not for u , we replace u back with $x^2 - 1$. This gives us

$$x^2 - 1 = 2 \quad \text{or} \quad x^2 - 1 = -1$$

$$x^2 = 3 \quad \text{or} \quad x^2 = 0$$

$$x = \pm\sqrt{3} \quad \text{or} \quad x = 0$$

Thus, the solution set is $\{-\sqrt{3}, 0, \sqrt{3}\}$.

- b. If we replace \sqrt{x} with, for example, a , then $x = a^2$, and the equation becomes

$$a^2 - 3a = 10, \quad / -10$$

which can be solved by factoring

$$a^2 - 3a - 10 = 0$$

$$(a + 2)(a - 5) = 0$$

$$a = -2 \quad \text{or} \quad a = 5$$

After replacing a back with \sqrt{x} , we have

$$\sqrt{x} = -2 \quad \text{or} \quad \sqrt{x} = 5.$$

The first equation, $\sqrt{x} = -2$, does not give us any solution as the square root cannot be negative. After squaring both sides of the second equation, we obtain $x = 25$. So, the solution set is **{25}**.

- c. The equation $\frac{1}{(a+2)^2} + \frac{1}{a+2} - 6 = 0$ can be solved as any other rational equation, by clearing the denominators via multiplying by the $LCD = (a + 2)^2$. However, it can also be seen as a quadratic equation as soon as we replace $\frac{1}{a+2}$ with, for example, x . By doing so, we obtain

$$x^2 + x - 6 = 0,$$

which after factoring

$$(x + 3)(x - 2) = 0,$$

gives us

$$x = -3 \quad \text{or} \quad x = 2$$

Remember to use a different letter than the variable in the original equation.

Again, since we need to solve the original equation for a , we replace x back with $\frac{1}{a+2}$.

This gives us

$$\frac{1}{a+2} = -3 \quad \text{or} \quad \frac{1}{a+2} = 2 \quad / \text{ take the reciprocal of both sides}$$

$$a + 2 = \frac{1}{-3} \quad \text{or} \quad a + 2 = \frac{1}{2} \quad / -2$$

$$a = -\frac{7}{3} \quad \text{or} \quad a = -\frac{3}{2}$$

Since both values are in the domain of the original equation, which is $\mathbb{R} \setminus \{0\}$, then the

solution set is $\{-\frac{7}{3}, -\frac{3}{2}\}$.

Solving Formulas

When solving formulas for a variable that appears in the second power, we use the same strategies as in solving quadratic equations. For example, we may use the square root property or the quadratic formula.

Example 2 ▶ Solving Formulas for a Variable that Appears in the Second Power

Solve each formula for the given variable.

a. $E = mc^2$, for c

b. $N = \frac{k^2 - 3k}{2}$, for k

Solution ▶

- a. To solve for c , first, we reverse the multiplication by m via the division by m . Then, we reverse the operation of squaring by taking the square root of both sides of the equation.

$$E = mc^2 \quad / \div m$$

$$\frac{E}{m} = c^2$$

Then, we reverse the operation of squaring by taking the square root of both sides of the equation. So, we have

$$\sqrt{\frac{E}{m}} = \sqrt{c^2},$$

Remember that
 $\sqrt{c^2} = |c|$, so we
 use the \pm sign in
 place of $| |$.

and therefore

$$c = \pm \sqrt{\frac{E}{m}}$$

- b. Observe that the variable k appears in the formula $N = \frac{k^2 - 3k}{2}$ in two places. Once in the first and once in the second power of k . This means that we can treat this formula as a quadratic equation with respect to k and solve it with the aid of the quadratic formula. So, we have

$$N = \frac{k^2 - 3k}{2} \quad / \cdot 2$$

$$2N = k^2 - 3k \quad / -2N$$

$$k^2 - 3k - 2N = 0$$

Substituting $a = 1$, $b = -3$, and $c = -2N$ to the quadratic formula, we obtain

$$k_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-2N)}}{2} = \frac{3 \pm \sqrt{9 + 8N}}{2}$$

Application Problems

Many application problems require solving quadratic equations. Sometimes this can be achieved via factoring, but often it is helpful to use the quadratic formula.

Example 3 ▶ Solving a Distance Problem with the Aid of the Quadratic Formula



Three towns A , B , and C are situated as shown in the accompanying figure. The roads at A form a right angle. The towns A and C are connected by a straight road as well. The distance from A to B is 7 kilometers less than the distance from B to C . The distance from A to C is 20 km. Approximate the remaining distances between the towns up to the tenth of a kilometer.

Solution ▶ Since the roads between towns form a right triangle, we can employ the Pythagorean equation

$$AC^2 = AB^2 + BC^2$$

Suppose that $BC = x$. Then $AB = x - 7$, and we have

$$20^2 = (x - 7)^2 + x^2$$

$$400 = x^2 - 14x + 49 + x^2$$

$$2x^2 - 14x - 351 = 0$$

Applying the quadratic formula, we obtain

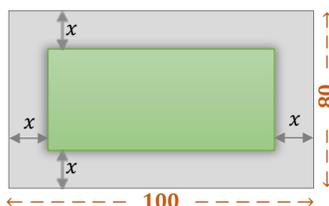
$$x_{1,2} = \frac{14 \pm \sqrt{14^2 + 4 \cdot 2 \cdot 351}}{4} = \frac{14 \pm \sqrt{196 + 2808}}{4} = \frac{14 \pm \sqrt{3004}}{4} \approx 17.2 \text{ or } -10.2$$

Since x represents a distance, it must be positive. So, the only solution is $x \approx 17.2$ km. Thus, the distance $BC \approx 17.2$ km and hence $AB \approx 17.2 - 7 = 10.2$ km.

Example 4 ▶ Solving a Geometry Problem with the Aid of the Quadratic Formula

A rectangular garden is 80 ft by 100 ft. Part of the garden is torn up to install a sidewalk of uniform width around it. The area of the new garden is $\frac{2}{3}$ of the old area. Approximately how wide is the sidewalk?

Solution ▶ To visualize the situation, we may draw a diagram as below.



Suppose x represents the width of the sidewalk. Then, the area of the new garden (the green area) can be expressed as $(100 - 2x)(80 - 2x)$. Since the green area is $\frac{2}{3}$ of the gray area (the original garden), we can form the equation

$$(100 - 2x)(80 - 2x) = \frac{2}{3}(100 \cdot 80)$$

To solve it, we may want to lower the coefficients by dividing both sides of the equation by 4 first. This gives us

$$\frac{\cancel{2}(50 - x) \cdot \cancel{2}(40 - x)}{\cancel{4}} = \frac{2}{3} \cdot \frac{(100 \cdot \cancel{80})}{\cancel{4}} \quad / \cdot 3$$

$$3(50 - x)(40 - x) = 4000$$

$$3(2000 - 90x + x^2) = 4000$$

$$3x^2 - 270x + 6000 = 4000 \quad / -2000$$

$$3x^2 - 270x + 2000 = 0,$$

which can be solved using the Quadratic Formula:

$$x_{1,2} = \frac{270 \pm \sqrt{270^2 - 4 \cdot 3 \cdot 2000}}{2 \cdot 3} = \frac{270 \pm \sqrt{48900}}{6} \approx \frac{270 \pm 221.13}{6}$$

$$= \begin{cases} \frac{491.13}{6} = 81.9 \\ \frac{48.87}{6} = 8.1 \end{cases}$$

The width x must be smaller than 40, so this value is too large to be considered.

Thus, the sidewalk is approximately **8.1** feet wide.

Example 5

Solving a Motion Problem That Requires the Use of the Quadratic Formula



The Columbia River flows at a rate of 2 mph for the length of a popular boating route. In order for a boat to travel 3 miles upriver and return in a total of 4 hours, how fast must the boat be able to travel in still water?

Solution

Suppose the rate of the boat moving in still water is r . Then, $r - 2$ represents the rate of the boat moving upriver and $r + 2$ represents the rate of the boat moving downriver. We can arrange these data in the table below.

	R	T	$= D$
upriver	$r - 2$	$\frac{3}{r - 2}$	3
downriver	$r + 2$	$\frac{3}{r + 2}$	3
total		4	

We fill the time-column by following the formula $T = \frac{D}{R}$.

By adding the times needed for traveling upriver and downriver, we form the rational equation

$$\frac{3}{r-2} + \frac{3}{r+2} = 4, \quad / \cdot (r^2 - 4)$$

which after multiplying by the $LCD = r^2 - 4$ becomes a quadratic equation.

$$\begin{aligned}
3(r + 2) + 3(r - 2) &= 4(r^2 - 4) \\
3r + 6 + 3r - 6 &= 4r^2 - 16 && / \cdot -6r \\
0 &= 4r^2 - 6r - 16 && / \div 2 \\
0 &= 2r^2 - 3r - 8
\end{aligned}$$

Using the Quadratic Formula, we have

$$r_{1,2} = \frac{3 \pm \sqrt{(-3)^2 + 4 \cdot 2 \cdot 8}}{2 \cdot 2} = \frac{3 \pm \sqrt{9 + 64}}{4} = \frac{3 \pm \sqrt{73}}{4} \approx \begin{cases} 2.9 \\ -1.4 \end{cases}$$

Since the rate cannot be negative, the boat moves in still water at approximately **2.9 mph**.

Example 6 ▶ Solving a Work Problem That Requires the Use of the Quadratic Formula

John can work through a stack of invoices in 1 hr less time than Bob can. Working together, they take 1.5 hr. To the nearest tenth of an hour, how long would it take each person working alone?

Solution ▶ Suppose the time needed for Bob to complete the job is t . Then, $t - 1$ represents the time needed for John to complete the same job. Now, we can arrange the given data in a table, as below.

	R	T	$= Job$
Bob	$\frac{1}{t}$	t	1
John	$\frac{1}{t-1}$	$t-1$	1
together	$\frac{2}{3}$	$\frac{3}{2}$	1

We fill the rate-column by following the formula $R = \frac{Job}{T}$.

By adding the rates of work for each person, we form the rational equation

$$\frac{1}{t} + \frac{1}{t-1} = \frac{2}{3}, \quad / \cdot 3t(t-1)$$

which after multiplying by the $LCD = 3t(t-1)$ becomes a quadratic equation.

$$\begin{aligned}
3(t-1) + 3t &= 2(t^2 - t) \\
3t - 3 + 3t &= 2t^2 - 2t && / -6t, +3 \\
0 &= 2t^2 - 8t + 3
\end{aligned}$$

Using the Quadratic Formula, we have

$$t_{1,2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{8 \pm \sqrt{64 - 24}}{4} = \frac{8 \pm \sqrt{40}}{4} \approx \begin{cases} 3.6 \\ 0.4 \end{cases}$$

Since the time needed for Bob cannot be shorter than 1 hr, we reject the 0.4 possibility. So, working alone, Bob requires approximately 3.6 hours, while John can do the same job in approximately 2.6 hours.

Example 7 ▶ **Solving a Projectile Problem Using a Quadratic Function**

If an object is projected upward from the top of a 120-ft building at 60 ft/sec, its position above the ground, h in feet, is given by the function $h(t) = -16t^2 + 60t + 120$, where t is the time in seconds after it was projected. To the nearest tenth of a second, when does the object hit the ground?

Solution ▶ The object hits the ground when the height h above the ground is equal to zero. So, we look for the solutions to the equation

$$h(t) = 0$$

which is equivalent to

$$-16t^2 + 60t + 120 = 0$$

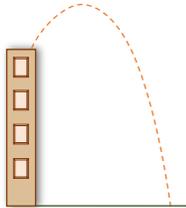
Before applying the Quadratic Formula, we may want to lower the coefficients by dividing both sides of the equation by -4 . So, we have

$$4t^2 - 15t - 30 = 0$$

and

$$t_{1,2} = \frac{15 \pm \sqrt{(-15)^2 + 4 \cdot 4 \cdot 30}}{2 \cdot 4} = \frac{15 \pm \sqrt{225 + 480}}{8} = \frac{15 \pm \sqrt{705}}{8} \approx \begin{cases} 41.6 \\ -1.4 \end{cases}$$

Thus, the object hits the ground in about 41.6 seconds.



Q.2 Exercises

Vocabulary Check Complete each blank with the most appropriate term or phrase from the given list: *approximate, exact, new, original, Quadratic Formula, quadratic in form, solutions.*

1. An equation is _____ if it can be rewritten in the form $au^2 + bu + c = 0$, where $a \neq 0$ and u represents an algebraic expression.
2. To solve an equation that is quadratic in form, rewrite it in terms of a _____ variable. Then, after solving for the new variable, go back to the _____ variable.
3. To solve a formula for a variable that appears in the first and second power, we use the _____.
4. When solving an application problem with a quadratic equation, check the proposed _____ against the information in the original problem.
5. When solving a quadratic equation, we usually leave the answer in the _____ form. When solving an application problem, we tend to _____ the answer, if needed.

Discussion Point

6. Discuss the following solution of the equation $\left(\frac{1}{x-2}\right)^2 - \frac{1}{x-2} - 2 = 0$:

Since this equation is quadratic in form, a student solved the related equation $a^2 - a - 2 = 0$ by factoring

$$(a - 2)(a + 1) = 0.$$

The student found the possible solutions, $a = 2$ and $a = -1$. Since 2 is not in the domain of the original equation, the student gave the final solution set as $\{-1\}$.

Solve each equation by treating it as a quadratic in form.

- | | | |
|---|---|--------------------------------|
| 7. $x^4 - 6x^2 + 9 = 0$ | 8. $x^8 - 29x^4 + 100 = 0$ | 9. $x - 10\sqrt{x} + 9 = 0$ |
| 10. $2x - 9\sqrt{x} + 4 = 0$ | 11. $y^{-2} - 5y^{-1} - 36 = 0$ | 12. $2a^{-2} + a^{-1} - 1 = 0$ |
| 13. $(1 + \sqrt{t})^2 + (1 + \sqrt{t}) - 6 = 0$ | 14. $(2 + \sqrt{x})^2 - 3(2 + \sqrt{x}) - 10 = 0$ | |
| 15. $(x^2 + 5x)^2 + 2(x^2 + 5x) - 24 = 0$ | 16. $(t^2 - 5t)^2 - 4(t^2 - 5t) + 3 = 0$ | |
| 17. $x^{\frac{2}{3}} - 4x^{\frac{1}{3}} - 5 = 0$ | 18. $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 8 = 0$ | |
| 19. $1 - \frac{1}{2p+1} - \frac{1}{(2p+1)^2} = 0$ | 20. $\frac{2}{(u+2)^2} + \frac{1}{u+2} = 3$ | |
| 21. $\left(\frac{x+3}{x-3}\right)^2 - \left(\frac{x+3}{x-3}\right) = 6$ | 22. $\left(\frac{y^2-1}{y}\right)^2 - 4\left(\frac{y^2-1}{y}\right) - 12 = 0$ | |

Solve each formula for the indicated variable.

- | | | |
|--|--|--|
| 23. $F = \frac{mv^2}{r}$, for v | 24. $V = \pi r^2 h$, for r | 25. $A = 4\pi r^2$, for r |
| 26. $V = \frac{1}{3}S^2 h$, for s | 27. $F = \frac{Gm_1 m_2}{r^2}$, for r | 28. $N = \frac{kq_1 q_2}{s^2}$, for s |
| 29. $a^2 + b^2 = c^2$, for b | 30. $I = \frac{703W}{H^2}$, for H | 31. $A = \pi r^2 + \pi r s$, for r |
| 32. $A = 2\pi r^2 + 2\pi r h$, for r | 33. $s = v_0 t + \frac{gt^2}{2}$, for t | 34. $t = \frac{a}{\sqrt{a^2 + b^2}}$, for a |
| 35. $P = \frac{A}{(1+r)^2}$, for r | 36. $P = EI - RI^2$, for I | 37. $s(6s - t) = t^2$, for s |
| 38. $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, for v , assuming that $c > 0$ and $m > 0$ | | |
| 39. $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, for c , assuming that $v > 0$ and $m > 0$ | | |
| 40. $p = \frac{E^2 R}{(r+R)^2}$, for E , assuming that $(r+R) > 0$ | | |

41. For over 2000 yr, the proportions of a “golden” rectangle have been considered visually appealing. A rectangle of width w and length l is considered “golden” if

$$\frac{w}{l} = \frac{l}{w + l}$$

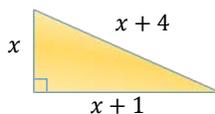
Solve for l .

Concept Check Answer each question.

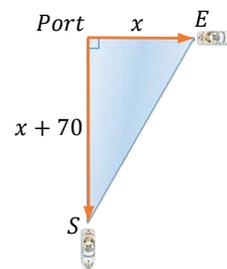
42. A boat goes r mph in still water, and the rate of the current is c mph.
- What is the rate of the boat when it travels upstream?
 - What is the rate of the boat when it travels downstream?
43. a. If it takes n hours to grade a set of papers, what is the grader’s rate in the amount of job completed per hour?
 b. How much of the job will the grader do in h hr?

Solve each problem.

44. Find the lengths of the sides of the triangle.

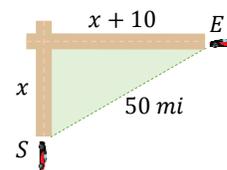


45. Two ships leave port at the same time, one heading due south and the other heading due east. Several hours later, they are 170 mi apart. If the ship traveling south traveled 70 mi farther than the other ship, how many miles did they each travel?
46. The diagonal of a rectangular rug measures 26 ft, and the length is 4 ft more than twice the width. Find the length and width of the rug.
47. The hypotenuse of a right triangle is 25 m long. The length of one leg is 17 m less than the other. Find the lengths of the legs.

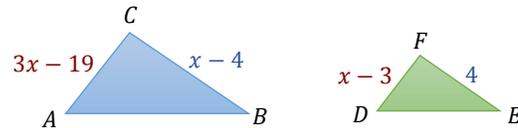


48. A 13-ft ladder is leaning against a house. The distance from the bottom of the ladder to the house is 7 ft less than the distance from the top of the ladder to the ground. How far is the bottom of the ladder from the house?

49. Two cars leave an intersection, one traveling south and one traveling east, as shown in the figure to the right. After 1 hour the two cars are 50 miles apart and the car traveling east has traveled 10 miles farther than the car traveling south. How far did each car travel?



50. The width of a rectangular computer screen is 3 inches more than its height. If the area of the screen is 154 square inches, find its dimensions.
51. The length of an American flag that is displayed at a government office is 3 inches less than twice its width. The area is 1710 in². Find the length and the width of the flag.
52. The length of a rectangle is twice the width. The area is 328 cm². Find the exact length and width.
53. Recall that corresponding sides of similar triangles are proportional. The diagram below shows two similar triangles, $\triangle ABC$ and $\triangle DEF$. Given the information in the diagram, find the length AC . Check all possible solutions. *Caution: The triangles are not drawn to scale here.*



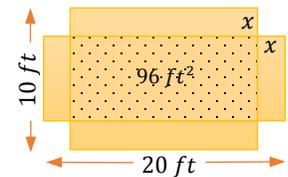
Analytic Skills Solve each problem.

54. Ludmila wants to buy a rug for a room that is 20 ft long and 15 ft wide. She wants to leave a strip of uncovered flooring of a uniform width around the edges of the room. How wide would a strip be if she bought a rug with an area of 234 ft^2 ?



55. A rectangular flower garden in a park is 30 feet wide and 40 feet long. A sidewalk of uniform width is being planned around the perimeter of the garden. The gardener has enough money to install 624 square feet of cement sidewalk. Find the width of the sidewalk.

56. An open box is to be made from a 10-ft by 20-ft rectangular piece of cardboard by cutting a square from each corner. The area of the bottom of the box is to be 96 ft^2 . What is the length of the sides of the squares that are cut from the corners?



57. The outside of a picture frame measures 13 cm by 20 cm, and 80 cm^2 of the picture shows. Find the width of the frame.
58. A rectangle has a length 2 m less than twice its width. When 5 m are added to the width, the resulting figure is a square with an area of 144 m^2 . Find the dimensions of the original rectangle.
59. A rectangular piece of sheet metal is 2 inches longer than it is wide. A square piece 3 inches on a side is cut from each corner. The sides are then turned up to form an uncovered box of volume 765 in^3 . Find the dimensions of the original piece of metal.
60. Karin's motorcycle traveled 300 mi at a certain speed. Had she gone 10 mph faster, she could have made the trip in 1 hr less time. Find her speed.
61. A turbo-jet flies 50 mph faster than a super-prop plane. If a turbo-jet goes 2000 mi in 3 hr less time than it takes the super-prop to go 2800 mi, find the speed of each plane.
62. A Cessna flies 600 mi. A Beechcraft flies 1000 mi at a speed that is 50 mph faster but takes 1 hr longer. If the planes do not fly slower than 200 mph, find the speed of each plane.
63. On a sales trip, Gail drives the 600 mi to Richmond. The return trip is made at a speed that is 10 mph slower. Total travel time for the round trip is 22 hr. How fast did Gail travel on each part of the trip?
64. The current in a typical Mississippi River shipping route flows at a rate of 4 mph. In order for a barge to travel 24 mi upriver and then return in a total of 5 hr, approximately how fast must the barge be able to travel in still water?



65. A camper paddles a canoe 2 miles downstream in a river that has a 2-mile-per-hour current. To return to camp, the canoeist travels upstream on a different branch of the river. It is 4 miles long and has a 1-mile-per-hour current. The total trip (both ways) takes 3 hours. Find the approximate speed of the canoe in still water.

66. One airplane flies east from an airport at an average speed of m mph. At the same time, a second airplane flies south from the same airport at an average speed that is 42 mph faster than the airplane traveling east. Thirty minutes after the two airplanes leave, they are 315 mi apart. Find the rate of each airplane to the nearest mile per hour.
67. A pilot flies 500 miles against a 20-mile-per-hour wind. On the next day, the pilot flies back home with a 10-mile-per-hour tailwind. The total trip (both ways) takes 4 hours. Approximate the speed of the airplane without a wind.
68. Working together, two people can clean an office building in 5 hr. One person is new to the job and would take 2 hr longer than the other person to clean the building alone. To the nearest tenth, how long would it take the new worker to clean the building alone?
69. Working together, two people can cut a large lawn in 2 hr. One person can do the job alone in 1 hr less time than the other. To the nearest tenth, how long would it take the faster worker to do the job?
70. Rusty and Nancy Brauner are planting flats of spring flowers. Working alone, Rusty would take 2 hr longer than Nancy to plant the flowers. Working together, they do the job in 12 hr. Approximately how long would it have taken each person working alone?
-  71. At Pizza Perfect, Ron can make 100 large pizza crusts in 1.2 hr less than Chad. Together they can do the job in 1.8 hr. To the nearest tenth, how long does it take each to do the job alone?
72. Two pipes together can fill a tank in 2 hr. One of the pipes, used alone, takes 3 hr longer than the other to fill the tank. How long would each pipe take to fill the tank alone?
73. A washing machine can be filled in 6 min if both the hot and cold water taps are fully opened. Filling the washer with hot water alone takes 9 min longer than filling it with cold water alone. How long does it take to fill the washer with cold water?
74. The height h of a stone t sec after it is thrown straight downward with an initial velocity of 20 feet per second from a bridge 800 ft above the ground is given by the equation $h = -16t^2 - 20t + 800$. When is the stone 300 ft above the ground?
75. A software company's weekly profit, P (in dollars), for selling x units of a new video game can be determined by the equation $P = -0.05x^2 + 48x - 100$. What is the smaller of the two numbers of units that must be sold in order to make a profit of \$9000?
76. The formula $A = P(1 + r)^2$ gives the amount A in dollars that P dollars will grow to in 2 yr at interest rate r (where r is given as a decimal), using compound interest. What interest rate will cause \$2000 to grow to \$2142.45 in 2 yr?

