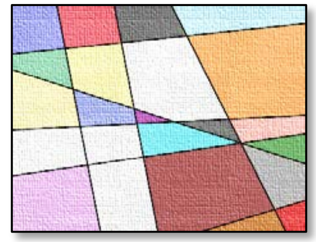
Systems of Linear Equations



As stated in *Section G1, Definition 1.1*, a linear equation in two variables is an equation of the form $Ax + By = C$, where A and B are not both zero. Such an equation has a line as its graph. Each point of this line is a solution to the equation in the sense that the coordinates of such a point satisfy the equation. So, there are infinitely many ordered pairs (x, y) satisfying the equation. Although analysis of the relation between x and y is instrumental in some problems, many application problems call for a particular, single solution. This occurs when, for example, x and y are required to satisfy an additional linear equation whose graph intersects the original line. In such a case, the solution to both equations is the point at which the lines intersect. Generally, to find unique values for **two** given **variables**, we need a system of **two equations** in these variables. In this section, we discuss several methods for solving systems of two linear equations.

E.1 Systems of Linear Equations in Two Variables

Any collection of equations considered together is called a **system of equations**. For example, a system consisting of two equations, $x + y = 5$ and $4x - y = 10$, is written as

$$\begin{cases} x + y = 5 \\ 4x - y = 10 \end{cases}$$

Since the equations in the system are linear, the system is called a **linear system of equations**.

Definition 1.1 ► A **solution** of a system of two equations in two variables, x and y, is any ordered pair (x, y) satisfying both equations of the system.

A **solution set** of a system of two linear equations in two variables, x and y, is the set of all possible solutions (x, y).

Note: The two variables used in a system of two equations can be denoted by any two different letters. In such case, to construct an ordered pair, we follow an alphabetical order. For example, if the variables are p and q, the corresponding ordered pair is (p, q), as p appears in the alphabet before q. This also means that a corresponding system of coordinates has the horizontal axis denoted as p-axis and the vertical axis denoted as q-axis.

Example 1 ► **Deciding Whether an Ordered Pair Is a Solution**

Decide whether the ordered pair $(3, 2)$ is a solution of the given system.

a. $\begin{cases} x + y = 5 \\ 4x - y = 10 \end{cases}$ **b.** $\begin{cases} m + 2n = 7 \\ 3m - n = 6 \end{cases}$

Solution ► **a.** To check whether the pair $(3, 2)$ is a solution, we let $x = 3$ and $y = 2$ in both equations of the system and check whether these equations are true. Since both equations,

$$\begin{aligned} 3 + 2 &= 5 \\ 5 &= 5 \; ✓ \end{aligned} \quad \text{and} \quad \begin{aligned} 4 \cdot 3 - 2 &= 10 \\ 10 &= 10, \; ✓ \end{aligned}$$

are true, then the pair $(3, 2)$ is a solution to the system.

b. First, we notice that alphabetically, m is before n. So, we let $m = 3$ and $n = 2$ and substitute these values into both equations.

$$3 + 2 \cdot 2 = 7 \qquad\qquad 3 \cdot 3 - 2 = 6$$
$$7 = 7 \qquad \text{but} \qquad 7 = 6$$

Since the pair $(3, 2)$ is not a solution of the second equation, it is not a solution to the whole system.

Solving Systems of Linear Equations by Graphing

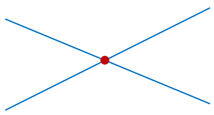


Figure 1a

Solutions to a system of two linear equations are all the ordered pairs that satisfy both equations. If an ordered pair satisfies an equation, then such a pair belongs to the graph of this equation. This means that the solutions to a system of two linear equations are the points that belong to both graphs of these lines. So, to solve such system, we can graph each line and take the common points as solutions.

How many solutions can a linear system of two equations have?

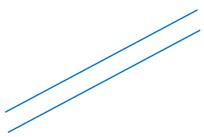


Figure 1b

There are three possible arrangements of two lines in a plane. The lines can **intersect** each other, be **parallel**, or be the **same**.

1. If a system of equations corresponds to the pair of **intersecting** lines, it has exactly **one solution**. The solution set consists of the **intersection point**, as shown in *Figure 1a*.

2. If a system of equations corresponds to the pair of **parallel** lines, it has **no solutions**. The solution set is empty, as shown in *Figure 1b*.

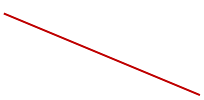


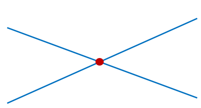
Figure 1c

3. If a system of equations corresponds to the pair of the **same** lines, it has **infinitely many solutions**. The solution set consists of all the **points of the line**, as shown in *Figure 1c*.

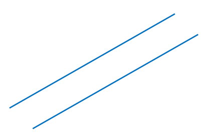
Definition 1.2 ► A linear system is called **consistent** if it has **at least one solution**. Otherwise, the system is **inconsistent**.

A linear system of two equations is called **independent** if the two lines are different. Otherwise, the system is **dependent**.

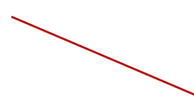
Here is the classification of systems corresponding to the following graphs:



consistent
independent



inconsistent
independent



consistent
dependent

Example 2 ► **Solving Systems of Linear Equations by Graphing**

Solve each system by graphing and classify it as *consistent*, *inconsistent*, *dependent* or *independent*.

a. $\begin{cases} 3p + q = 5 \\ p - 2q = 4 \end{cases}$ **b.** $\begin{cases} 3y - 2x = 6 \\ 4x - 6y = -12 \end{cases}$ **c.** $\begin{cases} f(x) = -\frac{1}{2}x + 3 \\ 2g(x) + x = -4 \end{cases}$

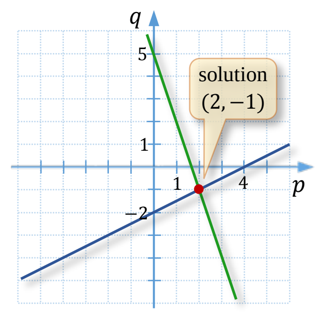
Solution ► **a.** To graph the first equation, it is convenient to use the slope-intercept form,

$$q = -3p + 5.$$

To graph the second equation, it is convenient to use the p- and q-intercepts, $(4, 0)$ and $(0, -2)$.

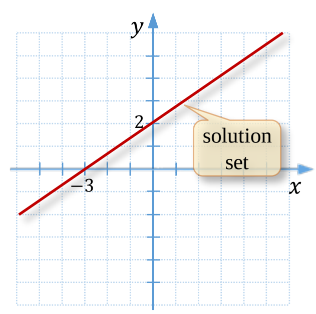
The first equation is graphed in green and the second – in blue. The intersection point is at $(\mathbf{2}, -\mathbf{1})$, which is the **only solution** of the system.

The system is **consistent**, as it has a solution, and **independent**, as the lines are different.



b. Notice that when using the x- and y-intercept method of graphing, both equations have x-intercepts equal to $(-3, 0)$ and y-intercepts equal to $(0, 2)$. So, both equations represent the same line. Therefore the solution set to this system consists of all points of the line $3y - 2x = 6$. We can record this set of points with the use of set-builder notation as $\{(\boldsymbol{x}, \boldsymbol{y}) | \mathbf{3}\boldsymbol{y} - \mathbf{2}\boldsymbol{x} = \mathbf{6}\}$, and state that the system has **infinitely many solutions**.

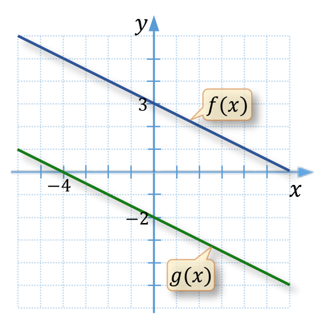
The system is **consistent**, as it has solutions, and **dependent**, as both lines are the same.



c. We plan to graph both functions, f and g, on the same grid. Function f is already given in the slope-intercept form, which is convenient for graphing. To graph function g, we can either use the x- and y-intercept method or solve the equation for $g(x)$ and use the slope-intercept method. So, we have

$$2g(x) = -x - 4$$

$$g(x) = -\tfrac{1}{2}x - 2$$



After graphing both functions, we observe that the two lines are parallel because they have the same slope. Having different y-intercepts, the lines do not have any common points. Therefore, the system has **no solutions**.

Such a system is **inconsistent**, as it has no solutions, and **independent**, as the lines are different.

Solving a system of equations by graphing, although useful, is not always a reliable method. For example, if the solution is an ordered pair of fractional numbers, we may have a hard time to read the exact values of these numbers from the graph. Luckily, a system of equations can be solved for exact values using algebra. Below, two algebraic methods for solving systems of two equations, referred to as substitution and elimination, are shown.

Solving Systems of Linear Equations by the Substitution Method

In the substitution method, as shown in *Example 3* below, we eliminate a variable from one equation by substituting an expression for that variable from the other equation. This method is particularly suitable for solving systems in which one variable is either already isolated or is easy enough to isolate.

Example 3 ► **Solving Systems of Linear Equations by Substitution**

Solve each system by substitution.

a. $\begin{cases} x = y + 1 \\ x + 2y = 4 \end{cases}$ **b.** $\begin{cases} 3a - 2b = 6 \\ 6a + 4b = -20 \end{cases}$

Solution ► **a.** Since x is already isolated in the first equation, $x = y + 1$, we replace x by $y + 1$ in the second equation, $x = y + 1$. Thus,

$$(y + 1) + 2y = 4 \qquad /-1$$

which after solving for y, gives us

$$3y = 3 \qquad /\div 3$$

$$\boldsymbol{y} = \mathbf{1}$$

Then, we substitute the value $y = 1$ back into the first equation, and solve for x. This gives us $\boldsymbol{x} = 1 + 1 = \mathbf{2}$.

One can check that $x = 2$ and $y = 1$ satisfy both of the original equations. So, the solution set of this system is $\{(\mathbf{2}, \mathbf{1})\}$.

b. To use the substitution method, we need to solve one of the equations for one of the variables, whichever is easier. Out of the coefficients by the variables, -2 seems to be the easiest coefficient to work with. So, let us solve the first equation, $3a - 2b = 6$, for b.

$$3a - 2b = 6 \qquad /-3a$$

$$-2b = -3a + 6 \qquad /\div(-2)$$

substitution equation

$$b = \frac{3}{2}a - 3$$

Then, substitute the expression $\frac{3}{2}a - 3$ to the second equation, $6a + 4b = -20$, for b. So, we obtain

$$6b + 4\left(\frac{3}{2}a - 3\right) = -20,$$

which can be solved for a:

equation in one variable

$$6a + 6a - 12 = -20 \qquad /+12$$

$$12a = -8 \qquad /\div 12$$

$$a = -\frac{8}{12}$$
$$\boldsymbol{a = -\frac{2}{3}}$$

Then, we plug $a = -\frac{2}{3}$ back into the substitution equation $b = \frac{3}{2}a - 3$ to find the b-value. This gives us

$$\boldsymbol{b} = \frac{3}{2}\left(-\frac{2}{3}\right) - 3 = -1 - 3 = \boldsymbol{-4}$$

To check that the values $a = -\frac{2}{3}$ and $b = -4$ satisfy both equations of the system, we substitute them into each equation, and simplify each side. Since both equations,

$$3\left(-\frac{2}{3}\right) - 2(-4) = 6$$
$$-2 + 8 = 6$$
$$6 = 6 \checkmark$$

and

$$6\left(-\frac{2}{3}\right) + 4(-4) = -20$$
$$-4 - 16 = -20$$
$$-20 = -20, \checkmark$$

are satisfied, the solution set of this system is $\left\{\left(-\frac{2}{3}, -4\right)\right\}$.

Summary of Solving Systems of Linear Equations by Substitution

Step 1	**Solve one of the equations for one of the variables.** Choose to solve for the variable with the easiest coefficient to work with. The obtained equation will be referred to as the **substitution equation**.
Step 2	**Plug the substitution equation into the other equation.** The result should be an equation with just one variable.
Step 3	**Solve** the resulting equation to find the value of the variable.
Step 4	**Find the value of the other variable** by substituting the result from Step 3 into the substitution equation from Step 1.
Step 5	**Check** if the variable values satisfy both of the original equations. Then **state the solution set** by listing the ordered pair(s) of numbers.

Solving Systems of Linear Equations by the Elimination Method

Another algebraic method, the **elimination method**, involves combining the two equations in a system so that one variable is eliminated. This is done using the addition property of equations.

Recall: If $\boldsymbol{a = b}$ and $\boldsymbol{c = d}$, then $\boldsymbol{a + c = b + d}$.

Example 6 ► **Solving Systems of Linear Equations by Elimination**

Solve each system by elimination.

a. $\begin{cases} r + 2s = 3 \\ 3r - 2s = 5 \end{cases}$

b. $\begin{cases} 2x + 3y = 6 \\ 3x + 5y = -2 \end{cases}$

Solution ► **a.** Notice that the equations contain opposite terms, $2s$ and $-2s$. Therefore, if we add these equations, side by side, the s-variable will be eliminated. So, we obtain

$$+\begin{cases} r + 2s = 3 \\ 3r - 2s = 5 \end{cases}$$

$$4r = 8$$

$$r = 2$$

Now, since the r-value is already known, we can substitute it to one of the equations of the system to find the s-value. Using the first equation, we obtain

$$2 + 2s = 3$$

$$2s = 1$$

$$s = \frac{1}{2}$$

One can check that the values $r = 2$ and $s = \frac{1}{2}$ make both equations of the original system true. Therefore, the pair $(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}})$ is the solution of this system. We say that the solution set is $\left\{(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}})\right\}$.

b. First, we choose which variable to eliminate. Suppose we plan to remove the x-variable. To do this, we need to transform the equations in such a way that the coefficients in the x-terms become opposite. This can be achieved by multiplying, for example, the first equation by 3 and the second equation, by -2.

$$\begin{cases} 2x + 3y = 6 & /\cdot 3 \\ 3x + 5y = -2 & /\cdot (-2) \end{cases}$$

$$+\begin{cases} 6x + 9y = 18 \\ -6x - 10y = 4 \end{cases}$$

Then, we add the two equations, side by side,

$$-y = 22 \qquad /\cdot (-1)$$

and solve the resulting equation for y,

$$y = -22.$$

To find the x-value, we substitute $y = -22$ to one of the original equations. Using the first equation, we obtain

$$2x + 3(-22) = 6$$

$$2x - 66 = 6 \qquad /+66$$

$$2x = 72 \qquad /\div 2$$

$$x = 36$$

One can check that the values $x = 36$ and $y = -22$ make both equations of the original system true. Therefore, the solution of this system is the pair $(\mathbf{36}, \mathbf{-22})$. We say that the solution set is $\{(\mathbf{36}, \mathbf{-22})\}$.

Summary of Solving Systems of Linear Equations by Elimination

Step 1 **Write both equations in standard form $Ax + By = C$.** Keep A and B as integers by clearing any fractions, if needed.

Step 2 **Choose a variable to eliminate.**

Step 3 **Make the chosen variable's terms opposites** by multiplying one or both equations by appropriate numbers if necessary.

Step 4 **Eliminate a variable by adding the respective sides of the equations** and then solve for the remaining variable.

Step 5 **Find the value of the other variable** by substituting the result from Step 4 into either of the original equations and solve for the other variable.

Step 6 **Check** if the variable values satisfy both of the original equations. Then **state the solution set** by listing the ordered pair(s) of numbers.

Comparing Methods of Solving Systems of Equations

When deciding which method to use, consider the suggestions in the table below.

Method	Strengths	Weaknesses
Graphical	• **Visualization.** The solutions can be "seen" and approximated.	• **Inaccuracy.** When solutions involve numbers that are not integers, they can only be approximated. • **Grid limitations.** Solutions may not appear on the part of the graph drawn.
Substitution	• **Exact solutions.** • Most convenient to use when a variable has a **coefficient of 1**.	• **Computations.** Often requires extensive computations with fractions.
Elimination	• **Exact solutions.** • Most convenient to use when all **coefficients** by variables are **different than 1**.	• **Preparation.** The method requires that the coefficients by one of the variables are opposite.

Solving Systems of Linear Equations in Special Cases

As it was shown in solving linear systems of equations by graphing, some systems have no solution or infinitely many solutions. The next example demonstrates how to solve such systems algebraically.

Example 7 ► **Solving Inconsistent or Dependent Systems of Linear Equations**

Solve each system algebraically.

a. $\begin{cases} x + 3y = 4 \\ -2x - 6y = 3 \end{cases}$ **b.** $\begin{cases} 2x - y = 3 \\ 6x - 3y = 9 \end{cases}$

Solution ► **a.** When trying to eliminate one of the variables, we might want to multiply the first equation by 2. This, however, causes both variables to be eliminated, resulting in

parallel lines

$$+\begin{cases} 2x + 6y = 8 \\ -2x - 6y = 3 \end{cases}$$

$$0 = 11,$$

which is *never true*. This means that there is no ordered pair (x, y) that would make this equation true. Therefore, there is **no solution** to this system. The solution set is $\emptyset$. The system is **inconsistent**, so the equations must describe **parallel lines**.

b. When trying to eliminate one of the variables, we might want to multiply the first equation by 3. This, however, causes both variables to be eliminated and we obtain

same line

$$+\begin{cases} 6x - 3y = 9 \\ 6x - 3y = 9 \end{cases}$$

$$0 = 0,$$

which is *always true*. This means that any x-value together with its corresponding y-value satisfy the system. Therefore, there are **infinitely many solutions** to this system. These solutions are all points of one of the equations. Therefore, the solution set can be recorded in set-builder notation, as

Read: **the set of all ordered pairs (x, y), such that $2x - y = 3$**

$$\{(x, y) | 2x - y = 3\}$$

Since the equations of the system are equivalent, they represent the same line. So, the system is **dependent**.

Summary of Special Cases of Linear Systems

If both variables are eliminated when solving a linear system of two equations, then the solution sets are determined as follows.

Case 1 If the resulting statement is **true**, there are **infinitely many solutions**. The system is ***consistent***, and the equations are ***dependent***.

Case 2 If the resulting statement is **false**, there is **no solution**. The system is ***inconsistent***, and the equations are ***independent***.

Another way of determining whether a system of two linear equations is inconsistent or dependent is by examining slopes and y-intercepts in the two equations.

Example 8 **Using Slope-Intercept Form to Determine the Number of Solutions and the Type of System**

For each system, determine the number of solutions and classify the system without actually solving it.

a. $\begin{cases} \frac{1}{2}x = \frac{1}{8}y + \frac{1}{4} \\ 4x - y = -2 \end{cases}$

b. $\begin{cases} 2x + 5y = 6 \\ 0.4x + y = 1.2 \end{cases}$

Solution

a. First, let us clear the fractions in the first equation by multiplying it by 8,

$$\begin{cases} 4x = y + 2 \\ 4x - y = -2 \end{cases}, \qquad \begin{matrix} /-2 \\ /+y, +2 \end{matrix}$$

and then solve each equation for y.

parallel lines

$$\begin{cases} 4x - 2 = y \\ 4x + 2 = y \end{cases}$$

Then, observe that the slopes in both equations are the same and equal to 4, but the y-intercepts are different, -2 and 2. The same slopes tell us that the corresponding lines are **parallel** while different y-intercepts tell us that the two lines are **different**. So, the system has **no solution**, which means it is **inconsistent**, and the lines are **independent**.

b. We will start by solving each equation for y. So, we have

$$\begin{cases} 2x + 5y = 6 \\ 0.4x + y = 1.2 \end{cases} \qquad \begin{matrix} /-2x \\ /-0.4x \end{matrix}$$

$$\begin{cases} 5y = -2x + 6 \\ y = -0.4x + 1.2 \end{cases} \qquad /\div 5$$

same line

$$\begin{cases} y = -\frac{2}{5}x + \frac{6}{5} \\ y = -0.4x + 1.2 \end{cases}$$

Notice that $-\frac{2}{5} = -0.4$ and $\frac{6}{5} = 1.2$. Since the resulting equations have the same slopes and the same y-intercepts, they represent the same line. Therefore, the system has **infinitely many solutions**, which means it is **consistent**, and the lines are **dependent**.

E.1 Exercises

Vocabulary Check *Fill in each blank with the most appropriate term or phrase from the given list:* ***common, consistent, empty, equivalent, inconsistent, independent, infinitely many, no, one, opposite, ordered pairs, system.***

1. A collection of two or more equations is a _________ of equations.

2. The solution set of a linear system of two equations consists of all ___________ _________ whose coordinates satisfy both equations of the system. The solutions correspond to the ___________ points of the graphs of the equations.

3. A linear system of equations that has at least one solution is called ______________.

4. A dependent linear system of equations has _____________ __________ solutions.

5. A linear system of equations with no solutions is called _______________.

6. A consistent system of ________________ linear equations has exactly one solution.

7. If solving a system leads to a false statement such as $0 = 1$, the solution set is __________.

8. If solving a system leads to a true statement such as $0 = 0$, the system consists of _______________ equations.

9. If the lines of a system of two equations have different slopes, the system has exactly _____ solution(s).

10. If both lines of a system of two equations have the same slope and different y-intercepts, the system has _______ solution(s).

11. To solve systems of equations by elimination, we write an equivalent system in which coefficients of one variable are _______________ .

Concept Check *Decide whether the given ordered pair is a solution of the given system.*

12. $\begin{cases} x + y = 7 \\ x - y = 3 \end{cases}$; $(5, 2)$

13. $\begin{cases} x + y = 1 \\ 2x - 3y = -8 \end{cases}$; $(-1, 2)$

14. $\begin{cases} p + 3q = 1 \\ 5p - q = -9 \end{cases}$; $(-2, 1)$

15. $\begin{cases} 2a + b = 3 \\ a - 2b = -9 \end{cases}$; $(-1, 5)$

Solve each system of equations ***graphically****. Then, classify the system as* ***consistent*** *or* ***inconsistent*** *and the equations as* ***dependent*** *or* ***independent****.*

16. $\begin{cases} 3x + y = 5 \\ x - 2y = 4 \end{cases}$

17. $\begin{cases} 3x + 4y = 8 \\ x + 2y = 6 \end{cases}$

18. $\begin{cases} f(x) = x - 1 \\ g(x) = -2x + 5 \end{cases}$

19. $\begin{cases} f(x) = -\frac{1}{4}x + 1 \\ g(x) = \frac{1}{2}x - 2 \end{cases}$

20. $\begin{cases} y - x = 5 \\ 2x - 2y = 10 \end{cases}$

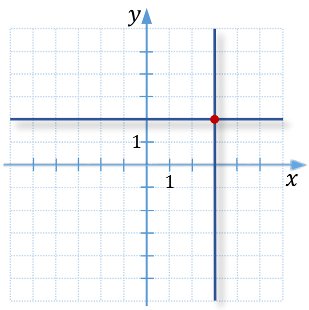
21. $\begin{cases} 6x - 2y = 2 \\ 9x - 3y = -1 \end{cases}$

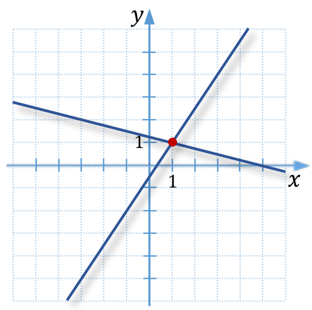
22. $\begin{cases} y = 3 - x \\ 2x + 2y = 6 \end{cases}$ **23.** $\begin{cases} 2x - 3y = 6 \\ 3y - 2x = -6 \end{cases}$ **24.** $\begin{cases} 2u + v = 3 \\ 2u = v + 7 \end{cases}$

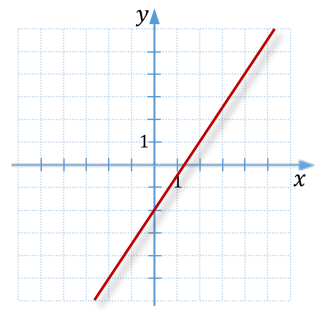
25. $\begin{cases} 2b = 6 - a \\ 3a - 2b = 6 \end{cases}$ **26.** $\begin{cases} f(x) = 2 \\ x = -3 \end{cases}$ **27.** $\begin{cases} f(x) = x \\ g(x) = -1.5 \end{cases}$

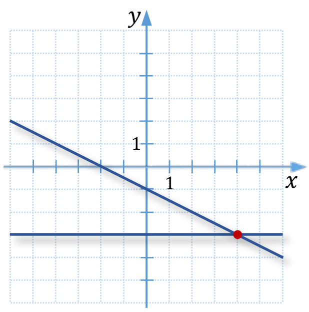
Concept Check

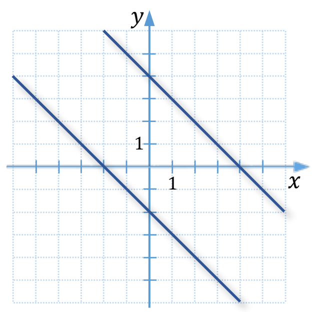
28. Classify each system **I** to **VI** as *consistent* or *inconsistent* and the equations as *dependent* or *independent*. Then, match it with the corresponding system of equations **A** to **F**.

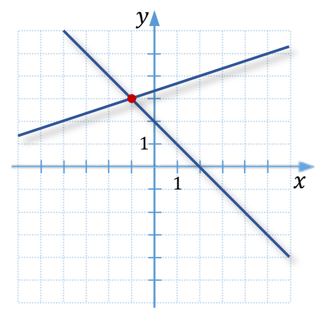
I


II


III


IV


V


VI


A $\begin{cases} 3y - x = 10 \\ x = -y + 2 \end{cases}$ **B** $\begin{cases} 9x - 6y = 12 \\ y = \frac{3}{2}x - 2 \end{cases}$ **C** $\begin{cases} 2y - 3x = -1 \\ x + 4y = 5 \end{cases}$

D $\begin{cases} x + y = 4 \\ y = -x - 2 \end{cases}$ **E** $\begin{cases} \frac{1}{2}x + y = -1 \\ y = -3 \end{cases}$ **F** $\begin{cases} x = 3 \\ y = 2 \end{cases}$

*Solve each system by **substitution**. If the system describes **parallel lines** or the **same line**, say so.*

29. $\begin{cases} y = 2x + 1 \\ 3x - 4y = 1 \end{cases}$ **30.** $\begin{cases} 5x - 6y = 23 \\ x = 6 - 3y \end{cases}$ **31.** $\begin{cases} x + 2y = 3 \\ 2x + y = 5 \end{cases}$

32. $\begin{cases} 2y = 1 - 4x \\ 2x + y = 0 \end{cases}$ **33.** $\begin{cases} y = 4 - 2x \\ y + 2x = 6 \end{cases}$ **34.** $\begin{cases} y - 2x = 3 \\ 4x - 2y = -6 \end{cases}$

35. $\begin{cases} 4s - 2t = 18 \\ 3s + 5t = 20 \end{cases}$ **36.** $\begin{cases} 4p + 2q = 8 \\ 5p - 7q = 1 \end{cases}$ **37.** $\begin{cases} \frac{x}{2} + \frac{y}{2} = 5 \\ \frac{3x}{2} - \frac{2y}{3} = 2 \end{cases}$

38. $\begin{cases} \frac{x}{4} + \frac{y}{3} = 0 \\ \frac{x}{8} - \frac{y}{6} = 2 \end{cases}$ **39.** $\begin{cases} 1.5a - 0.5b = 8.5 \\ 3a + 1.5b = 6 \end{cases}$ **40.** $\begin{cases} 0.3u - 2.4v = -2.1 \\ 0.04u + 0.03v = 0.7 \end{cases}$

Solve each system by ***elimination****. If the system describes* ***parallel lines*** *or the* ***same line****, say so.*

41. $\begin{cases} x+y=20 \\ x-y=4 \end{cases}$

42. $\begin{cases} 6x+5y=-7 \\ -6x-11y=1 \end{cases}$

43. $\begin{cases} x-y=5 \\ 3x+2y=10 \end{cases}$

44. $\begin{cases} x-4y=-3 \\ -3x+5y=2 \end{cases}$

45. $\begin{cases} 2x+3y=1 \\ 3x-5y=-8 \end{cases}$

46. $\begin{cases} -2x+5y=14 \\ 7x+6y=-2 \end{cases}$

47. $\begin{cases} 2x+3y=1 \\ 4x+6y=2 \end{cases}$

48. $\begin{cases} 6x-10y=-4 \\ 5y-3x=7 \end{cases}$

49. $\begin{cases} 0.3x-0.2y=4 \\ 0.2x+0.3y=1 \end{cases}$

50. $\begin{cases} \frac{2}{3}x+\frac{1}{7}y=-11 \\ \frac{1}{7}x-\frac{1}{3}y=-10 \end{cases}$

51. $\begin{cases} 3a+2b=3 \\ 9a-8b=-2 \end{cases}$

52. $\begin{cases} 5m-9n=7 \\ 7n-3m=-5 \end{cases}$

Discussion Point

53. A linear system of two equations has the solutions $(1,3)$ and $(4,-2)$. Are there any other solutions? If "no", explain why not. If "yes", find another solution.

Write each equation in slope-intercept form and then tell how many solutions the system has. Do not actually solve.

54. $\begin{cases} -x+2y=8 \\ 4x-8y=1 \end{cases}$

55. $\begin{cases} 6x=-9y+3 \\ 2x=-3y+1 \end{cases}$

56. $\begin{cases} y-x=6 \\ x+y=6 \end{cases}$

Solve each system by the method of your choice.

57. $\begin{cases} 3x+y=-7 \\ x-y=-5 \end{cases}$

58. $\begin{cases} 3x-2y=0 \\ 9x+8y=7 \end{cases}$

59. $\begin{cases} 3x-5y=7 \\ 2x+3y=30 \end{cases}$

60. $\begin{cases} 2x+3y=10 \\ -3x+y=18 \end{cases}$

61. $\begin{cases} \frac{1}{6}x+\frac{1}{3}y=8 \\ \frac{1}{4}x+\frac{1}{2}y=30 \end{cases}$

62. $\begin{cases} \frac{1}{2}x-\frac{1}{8}y=-\frac{1}{4} \\ 4x-y=-2 \end{cases}$

63. $\begin{cases} a+4b=2 \\ 5a-b=3 \end{cases}$

64. $\begin{cases} 3a-b=7 \\ 2a+2b=5 \end{cases}$

65. $\begin{cases} 6\cdot f(x)=2x \\ -7x+15\cdot g(x)=10 \end{cases}$

Analytic Skills *Solve each system of linear equations. Assume that* ***a*** *and* ***b*** *represent nonzero constants.*

66. $\begin{cases} x+ay=1 \\ 2x+2ay=4 \end{cases}$

67. $\begin{cases} -ax+y=4 \\ ax+y=4 \end{cases}$

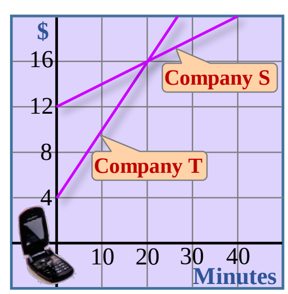
68. $\begin{cases} -ax+y=2 \\ ax+y=4 \end{cases}$

69. $\begin{cases} ax+by=2 \\ -ax+2by=1 \end{cases}$

70. $\begin{cases} 2ax-y=3 \\ y=5ax \end{cases}$

71. $\begin{cases} 3ax+2y=1 \\ -ax+y=2 \end{cases}$

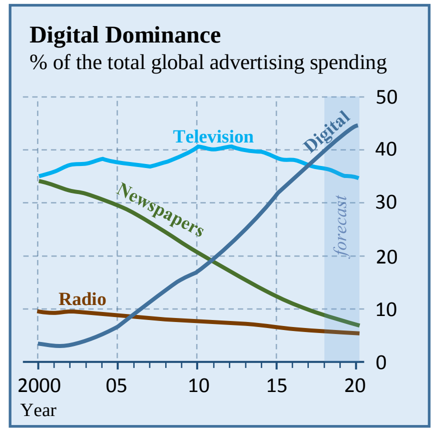
Refer to the accompanying graph to answer questions 63-64.



72. According to the graph, for how many air-time minutes do the two companies charge the same amount? Give an ordered pair of the form (minutes, dollars) to represent the situation.

73. For what range of air-time minutes is company T cheaper?

Refer to the accompanying graph to answer questions 65-68.



74. According to the predictions shown in the graph, what percent of the global advertising spending will be allocated to digital advertising in 2020? Give an ordered pair of the form (year, percent) to represent this information.

75. Estimate the year in which spending on digital advertising matches spending on television advertising.

76. Since when did spending on digital advertising exceed spending on advertising in newspapers? What was this spending as a percentage of global advertising spending at that time?

77. Since when did spending on digital advertising exceed spending on radio advertising? What was this spending as a percentage of global advertising spending at that time?