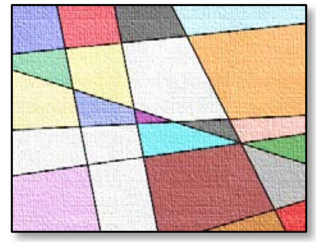
Systems of Linear Equations



As stated in *Section G1, Definition 1.1*, a linear equation in two variables is an equation of the form $Ax + By = C$, where A and B are not both zero. Such an equation has a line as its graph. Each point of this line is a solution to the equation in the sense that the coordinates of such a point satisfy the equation. So, there are infinitely many ordered pairs (x, y) satisfying the equation. Although analysis of the relation between x and y is instrumental in some problems, many application problems call for a particular, single solution. This occurs when, for example, x and y are required to satisfy an additional linear equation whose graph intersects the original line. In such a case, the solution to both equations is the point at which the lines intersect. Generally, to find unique values for **two** given **variables**, we need a system of **two equations** in these variables. In this section, we discuss several methods for solving systems of two linear equations.

E.1 Systems of Linear Equations in Two Variables

Any collection of equations considered together is called a **system of equations**. For example, a system consisting of two equations, $x + y = 5$ and $4x - y = 10$, is written as

$$\begin{cases} x + y = 5 \\ 4x - y = 10 \end{cases}$$

Since the equations in the system are linear, the system is called a **linear system of equations**.

Definition 1.1 ► A **solution** of a system of two equations in two variables, x and y, is any ordered pair (x, y) satisfying both equations of the system.

A **solution set** of a system of two linear equations in two variables, x and y, is the set of all possible solutions (x, y).

Note: The two variables used in a system of two equations can be denoted by any two different letters. In such case, to construct an ordered pair, we follow an alphabetical order. For example, if the variables are p and q, the corresponding ordered pair is (p, q), as p appears in the alphabet before q. This also means that a corresponding system of coordinates has the horizontal axis denoted as p-axis and the vertical axis denoted as q-axis.

Example 1 ► **Deciding Whether an Ordered Pair Is a Solution**

Decide whether the ordered pair $(3, 2)$ is a solution of the given system.

a. $\begin{cases} x + y = 5 \\ 4x - y = 10 \end{cases}$ **b.** $\begin{cases} m + 2n = 7 \\ 3m - n = 6 \end{cases}$

Solution ► **a.** To check whether the pair $(3, 2)$ is a solution, we let $x = 3$ and $y = 2$ in both equations of the system and check whether these equations are true. Since both equations,

$$\begin{aligned} 3 + 2 &= 5 \\ 5 &= 5 \checkmark \end{aligned} \qquad \text{and} \qquad \begin{aligned} 4 \cdot 3 - 2 &= 10 \\ 10 &= 10, \checkmark \end{aligned}$$

are true, then the pair $(3, 2)$ is a solution to the system.

b. First, we notice that alphabetically, m is before n. So, we let $m = 3$ and $n = 2$ and substitute these values into both equations.

$$3 + 2 \cdot 2 = 7$$
$$7 = 7$$

but

$$3 \cdot 3 - 2 = 6$$
$$7 = 6$$

Since the pair $(3, 2)$ is not a solution of the second equation, it is not a solution to the whole system.

Solving Systems of Linear Equations by Graphing

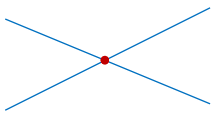


Figure 1a

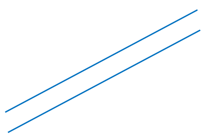


Figure 1b

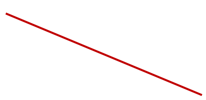


Figure 1c

Solutions to a system of two linear equations are all the ordered pairs that satisfy both equations. If an ordered pair satisfies an equation, then such a pair belongs to the graph of this equation. This means that the solutions to a system of two linear equations are the points that belong to both graphs of these lines. So, to solve such system, we can graph each line and take the common points as solutions.

How many solutions can a linear system of two equations have?

There are three possible arrangements of two lines in a plane. The lines can **intersect** each other, be **parallel**, or be the **same**.

1. If a system of equations corresponds to the pair of **intersecting** lines, it has exactly **one solution**. The solution set consists of the **intersection point**, as shown in *Figure 1a*.

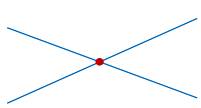
2. If a system of equations corresponds to the pair of **parallel** lines, it has **no solutions**. The solution set is empty, as shown in *Figure 1b*.

3. If a system of equations corresponds to the pair of the **same** lines, it has **infinitely many solutions**. The solution set consists of all the **points of the line**, as shown in *Figure 1c*.

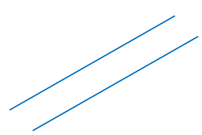
Definition 1.2 ► A linear system is called **consistent** if it has **at least one solution**. Otherwise, the system is **inconsistent**.

A linear system of two equations is called **independent** if the two lines are different. Otherwise, the system is **dependent**.

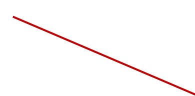
Here is the classification of systems corresponding to the following graphs:



consistent
independent



inconsistent
independent



consistent
dependent

Example 2 ► **Solving Systems of Linear Equations by Graphing**

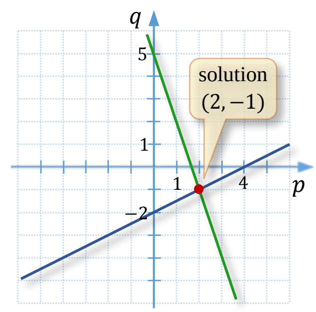
Solve each system by graphing and classify it as *consistent*, *inconsistent*, *dependent* or *independent*.

a. $\begin{cases} 3p + q = 5 \\ p - 2q = 4 \end{cases}$ **b.** $\begin{cases} 3y - 2x = 6 \\ 4x - 6y = -12 \end{cases}$ **c.** $\begin{cases} f(x) = -\frac{1}{2}x + 3 \\ 2g(x) + x = -4 \end{cases}$

Solution ► **a.** To graph the first equation, it is convenient to use the slope-intercept form,

$$q = -3p + 5.$$

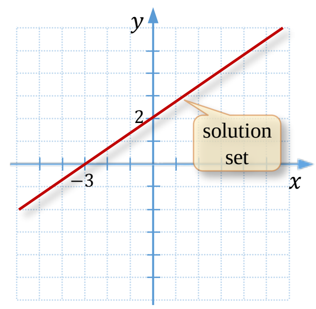
To graph the second equation, it is convenient to use the p- and q-intercepts, $(4, 0)$ and $(0, -2)$.



The first equation is graphed in green and the second – in blue. The intersection point is at $(\mathbf{2}, \mathbf{-1})$, which is the **only solution** of the system.

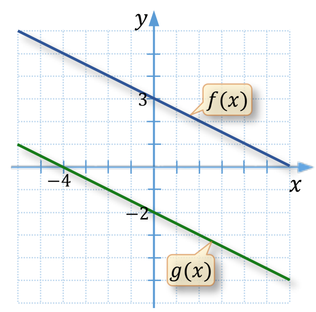
The system is **consistent**, as it has a solution, and **independent**, as the lines are different.

b. Notice that when using the x- and y-intercept method of graphing, both equations have x-intercepts equal to $(-3, 0)$ and y-intercepts equal to $(0, 2)$. So, both equations represent the same line. Therefore the solution set to this system consists of all points of the line $3y - 2x = 6$. We can record this set of points with the use of set-builder notation as $\{(\boldsymbol{x}, \boldsymbol{y}) | \mathbf{3}\boldsymbol{y} - \mathbf{2}\boldsymbol{x} = \mathbf{6}\}$, and state that the system has **infinitely many solutions**.



The system is **consistent**, as it has solutions, and **dependent**, as both lines are the same.

c. We plan to graph both functions, f and g, on the same grid. Function f is already given in the slope-intercept form, which is convenient for graphing. To graph function g, we can either use the x- and y-intercept method or solve the equation for $g(x)$ and use the slope-intercept method. So, we have



$$2g(x) = -x - 4$$

$$g(x) = -\frac{1}{2}x - 2$$

After graphing both functions, we observe that the two lines are parallel because they have the same slope. Having different y-intercepts, the lines do not have any common points. Therefore, the system has **no solutions**.

Such a system is **inconsistent**, as it has no solutions, and **independent**, as the lines are different.

Solving a system of equations by graphing, although useful, is not always a reliable method. For example, if the solution is an ordered pair of fractional numbers, we may have a hard time to read the exact values of these numbers from the graph. Luckily, a system of equations can be solved for exact values using algebra. Below, two algebraic methods for solving systems of two equations, referred to as substitution and elimination, are shown.

Solving Systems of Linear Equations by the Substitution Method

In the substitution method, as shown in *Example 3* below, we eliminate a variable from one equation by substituting an expression for that variable from the other equation. This method is particularly suitable for solving systems in which one variable is either already isolated or is easy enough to isolate.

Example 3 ► **Solving Systems of Linear Equations by Substitution**

Solve each system by substitution.

a. $\begin{cases} x = y + 1 \\ x + 2y = 4 \end{cases}$ **b.** $\begin{cases} 3a - 2b = 6 \\ 6a + 4b = -20 \end{cases}$

Solution ► **a.** Since x is already isolated in the first equation, $x = y + 1$, we replace x by $y + 1$ in the second equation, $x = y + 1$. Thus,

$$(y + 1) + 2y = 4 \qquad /-1$$

which after solving for y, gives us

$$3y = 3 \qquad /\div 3$$
$$\boldsymbol{y} = \mathbf{1}$$

Then, we substitute the value $y = 1$ back into the first equation, and solve for x. This gives us $\boldsymbol{x} = 1 + 1 = \mathbf{2}$.
One can check that $x = 2$ and $y = 1$ satisfy both of the original equations. So, the solution set of this system is $\{(\mathbf{2}, \mathbf{1})\}$.

b. To use the substitution method, we need to solve one of the equations for one of the variables, whichever is easier. Out of the coefficients by the variables, -2 seems to be the easiest coefficient to work with. So, let us solve the first equation, $3a - 2b = 6$, for b.

$$3a - 2b = 6 \qquad /-3a$$
$$-2b = -3a + 6 \qquad /\div(-2)$$

substitution equation

$$b = \frac{3}{2}a - 3$$

Then, substitute the expression $\frac{3}{2}a - 3$ to the second equation, $6a + 4b = -20$, for b. So, we obtain

$$6b + 4\left(\frac{3}{2}a - 3\right) = -20,$$

which can be solved for a:

equation in one variable

$$6a + 6a - 12 = -20 \qquad /+12$$
$$12a = -8 \qquad /\div 12$$

$$a = -\frac{8}{12}$$
$$\boldsymbol{a = -\frac{2}{3}}$$

Then, we plug $a = -\frac{2}{3}$ back into the substitution equation $b = \frac{3}{2}a - 3$ to find the b-value. This gives us

$$\boldsymbol{b} = \frac{3}{2}\left(-\frac{2}{3}\right) - 3 = -1 - 3 = \boldsymbol{-4}$$

To check that the values $a = -\frac{2}{3}$ and $b = -4$ satisfy both equations of the system, we substitute them into each equation, and simplify each side. Since both equations,

$$\begin{aligned} 3\left(-\frac{2}{3}\right) - 2(-4) &= 6 \\ -2 + 8 &= 6 \\ 6 &= 6 \checkmark \end{aligned} \qquad \text{and} \qquad \begin{aligned} 6\left(-\frac{2}{3}\right) + 4(-4) &= -20 \\ -4 - 16 &= -20 \\ -20 &= -20, \checkmark \end{aligned}$$

are satisfied, the solution set of this system is $\left\{\left(-\frac{2}{3}, -4\right)\right\}$.

Summary of Solving Systems of Linear Equations by Substitution

Step 1 **Solve one of the equations for one of the variables.** Choose to solve for the variable with the easiest coefficient to work with. The obtained equation will be referred to as the **substitution equation**.

Step 2 **Plug the substitution equation into the <u>other</u> equation.** The result should be an equation with just <u>one</u> variable.

Step 3 **Solve** the resulting equation to find the value of the variable.

Step 4 **Find the value of the other variable** by substituting the result from Step 3 into the substitution equation from Step 1.

Step 5 **Check** if the variable values satisfy both of the original equations. Then **state the solution set** by listing the ordered pair(s) of numbers.

Solving Systems of Linear Equations by the Elimination Method

Another algebraic method, the **elimination method**, involves combining the two equations in a system so that one variable is eliminated. This is done using the addition property of equations.

Recall: If $\boldsymbol{a = b}$ and $\boldsymbol{c = d}$, then $\boldsymbol{a + c = b + d}$.

Example 6 ► **Solving Systems of Linear Equations by Elimination**

Solve each system by elimination.

a. $\begin{cases} r + 2s = 3 \\ 3r - 2s = 5 \end{cases}$

b. $\begin{cases} 2x + 3y = 6 \\ 3x + 5y = -2 \end{cases}$

Solution ► **a.** Notice that the equations contain opposite terms, $2s$ and $-2s$. Therefore, if we add these equations, side by side, the s-variable will be eliminated. So, we obtain

$$+\begin{cases} r + 2s = 3 \\ 3r - 2s = 5 \end{cases}$$

$$4r = 8$$
$$r = 2$$

Now, since the r-value is already known, we can substitute it to one of the equations of the system to find the s-value. Using the first equation, we obtain

$$2 + 2s = 3$$
$$2s = 1$$
$$s = \frac{1}{2}$$

One can check that the values $r = 2$ and $s = \frac{1}{2}$ make both equations of the original system true. Therefore, the pair $\left(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}\right)$ is the solution of this system. We say that the solution set is $\left\{\left(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}\right)\right\}$.

b. First, we choose which variable to eliminate. Suppose we plan to remove the x-variable. To do this, we need to transform the equations in such a way that the coefficients in the x-terms become opposite. This can be achieved by multiplying, for example, the first equation by 3 and the second equation, by -2.

$$\begin{cases} 2x + 3y = 6 \\ 3x + 5y = -2 \end{cases} \qquad \begin{matrix} /\cdot 3 \\ /\cdot (-2) \end{matrix}$$

$$+\begin{cases} 6x + 9y = 18 \\ -6x - 10y = 4 \end{cases}$$

Then, we add the two equations, side by side,

$$-y = 22 \qquad /\cdot (-1)$$

and solve the resulting equation for y,

$$y = -22.$$

To find the x-value, we substitute $y = -22$ to one of the original equations. Using the first equation, we obtain

$$2x + 3(-22) = 6$$

$$2x - 66 = 6 \qquad /+66$$

$$2x = 72 \qquad /\div 2$$

$$x = 36$$

One can check that the values $x = 36$ and $y = -22$ make both equations of the original system true. Therefore, the solution of this system is the pair $(\mathbf{36}, \mathbf{-22})$. We say that the solution set is $\{(\mathbf{36}, \mathbf{-22})\}$.

Summary of Solving Systems of Linear Equations by Elimination

Step 1	**Write both equations in standard form $\boldsymbol{Ax + By = C}$.** Keep A and B as integers by clearing any fractions, if needed.
Step 2	**Choose a variable to eliminate.**
Step 3	**Make the chosen variable's terms opposites** by multiplying one or both equations by appropriate numbers if necessary.
Step 4	**Eliminate a variable by adding the respective sides of the equations** and then solve for the remaining variable.
Step 5	**Find the value of the other variable** by substituting the result from Step 4 into either of the original equations and solve for the other variable.
Step 6	**Check** if the variable values satisfy both of the original equations. Then **state the solution set** by listing the ordered pair(s) of numbers.

Comparing Methods of Solving Systems of Equations

When deciding which method to use, consider the suggestions in the table below.

Method	Strengths	Weaknesses
Graphical	• **Visualization.** The solutions can be "seen" and approximated.	• **Inaccuracy.** When solutions involve numbers that are not integers, they can only be approximated. • **Grid limitations.** Solutions may not appear on the part of the graph drawn.
Substitution	• **Exact solutions.** • Most convenient to use when a variable has a **coefficient of 1**.	• **Computations.** Often requires extensive computations with fractions.
Elimination	• **Exact solutions.** • Most convenient to use when all **coefficients** by variables are **different than 1**.	• **Preparation.** The method requires that the coefficients by one of the variables are opposite.

Solving Systems of Linear Equations in Special Cases

As it was shown in solving linear systems of equations by graphing, some systems have no solution or infinitely many solutions. The next example demonstrates how to solve such systems algebraically.

Example 7 ► **Solving Inconsistent or Dependent Systems of Linear Equations**

Solve each system algebraically.

a. $\begin{cases} x + 3y = 4 \\ -2x - 6y = 3 \end{cases}$ **b.** $\begin{cases} 2x - y = 3 \\ 6x - 3y = 9 \end{cases}$

Solution ► **a.** When trying to eliminate one of the variables, we might want to multiply the first equation by 2. This, however, causes both variables to be eliminated, resulting in

parallel lines

$$\begin{array}{r} \begin{cases} 2x + 6y = 8 \\ -2x - 6y = 3 \end{cases} \\ \hline 0 = 11, \end{array}$$

which is *never true*. This means that there is no ordered pair (x, y) that would make this equation true. Therefore, there is **no solution** to this system. The solution set is $\emptyset$. The system is **inconsistent**, so the equations must describe **parallel lines**.

b. When trying to eliminate one of the variables, we might want to multiply the first equation by 3. This, however, causes both variables to be eliminated and we obtain

same line

$$\begin{array}{r} \begin{cases} 6x - 3y = 9 \\ 6x - 3y = 9 \end{cases} \\ \hline 0 = 0, \end{array}$$

which is *always true*. This means that any x-value together with its corresponding y-value satisfy the system. Therefore, there are **infinitely many solutions** to this system. These solutions are all points of one of the equations. Therefore, the solution set can be recorded in set-builder notation, as

Read: the set of all ordered pairs (x, y), such that $2x - y = 3$

$$\{(\boldsymbol{x}, \boldsymbol{y}) | \mathbf{2}\boldsymbol{x} - \boldsymbol{y} = \mathbf{3}\}$$

Since the equations of the system are equivalent, they represent the same line. So, the system is **dependent**.

Summary of Special Cases of Linear Systems

If both variables are eliminated when solving a linear system of two equations, then the solution sets are determined as follows.

Case 1 If the resulting statement is **true**, there are **infinitely many solutions**. The system is ***consistent***, and the equations are ***dependent***.

Case 2 If the resulting statement is **false**, there is **no solution**. The system is ***inconsistent***, and the equations are ***independent***.

Another way of determining whether a system of two linear equations is inconsistent or dependent is by examining slopes and y-intercepts in the two equations.

Example 8 ► **Using Slope-Intercept Form to Determine the Number of Solutions and the Type of System**

For each system, determine the number of solutions and classify the system without actually solving it.

a. $\begin{cases} \frac{1}{2}x = \frac{1}{8}y + \frac{1}{4} \\ 4x - y = -2 \end{cases}$ **b.** $\begin{cases} 2x + 5y = 6 \\ 0.4x + y = 1.2 \end{cases}$

Solution ► **a.** First, let us clear the fractions in the first equation by multiplying it by 8,

$$\begin{cases} 4x = y + 2 \\ 4x - y = -2 \end{cases}, \qquad \begin{array}{l} /-2 \\ /+y, +2 \end{array}$$

and then solve each equation for y.

parallel lines

$$\begin{cases} 4x - 2 = y \\ 4x + 2 = y \end{cases}$$

Then, observe that the slopes in both equations are the same and equal to 4, but the y-intercepts are different, -2 and 2. The same slopes tell us that the corresponding lines are **parallel** while different y-intercepts tell us that the two lines are **different**. So, the system has **no solution**, which means it is **inconsistent**, and the lines are **independent**.

b. We will start by solving each equation for y. So, we have

$$\begin{cases} 2x + 5y = 6 \\ 0.4x + y = 1.2 \end{cases} \qquad \begin{array}{l} /-2x \\ /-0.4x \end{array}$$

$$\begin{cases} 5y = -2x + 6 \\ y = -0.4x + 1.2 \end{cases} \qquad /\div 5$$

same line

$$\begin{cases} y = -\frac{2}{5}x + \frac{6}{5} \\ y = -0.4x + 1.2 \end{cases}$$

Notice that $-\frac{2}{5} = -0.4$ and $\frac{6}{5} = 1.2$. Since the resulting equations have the same slopes and the same y-intercepts, they represent the same line. Therefore, the system has **infinitely many solutions**, which means it is **consistent**, and the lines are **dependent**.

E.1 Exercises

Vocabulary Check *Fill in each blank with the most appropriate term or phrase from the given list:* ***common, consistent, empty, equivalent, inconsistent, independent, infinitely many, no, one, opposite, ordered pairs, system.***

1. A collection of two or more equations is a _________ of equations.

2. The solution set of a linear system of two equations consists of all ___________ _________ whose coordinates satisfy both equations of the system. The solutions correspond to the ___________ points of the graphs of the equations.

3. A linear system of equations that has at least one solution is called ______________.

4. A dependent linear system of equations has _____________ __________ solutions.

5. A linear system of equations with no solutions is called _______________.

6. A consistent system of ________________ linear equations has exactly one solution.

7. If solving a system leads to a false statement such as $0 = 1$, the solution set is __________.

8. If solving a system leads to a true statement such as $0 = 0$, the system consists of _______________ equations.

9. If the lines of a system of two equations have different slopes, the system has exactly _____ solution(s).

10. If both lines of a system of two equations have the same slope and different y-intercepts, the system has _______ solution(s).

11. To solve systems of equations by elimination, we write an equivalent system in which coefficients of one variable are _______________ .

Concept Check *Decide whether the given ordered pair is a solution of the given system.*

12. $\begin{cases} x + y = 7 \\ x - y = 3 \end{cases}$; $(5, 2)$

13. $\begin{cases} x + y = 1 \\ 2x - 3y = -8 \end{cases}$; $(-1, 2)$

14. $\begin{cases} p + 3q = 1 \\ 5p - q = -9 \end{cases}$; $(-2, 1)$

15. $\begin{cases} 2a + b = 3 \\ a - 2b = -9 \end{cases}$; $(-1, 5)$

Solve each system of equations ***graphically****. Then, classify the system as* ***consistent*** *or* ***inconsistent*** *and the equations as* ***dependent*** *or* ***independent****.*

16. $\begin{cases} 3x + y = 5 \\ x - 2y = 4 \end{cases}$

17. $\begin{cases} 3x + 4y = 8 \\ x + 2y = 6 \end{cases}$

18. $\begin{cases} f(x) = x - 1 \\ g(x) = -2x + 5 \end{cases}$

19. $\begin{cases} f(x) = -\frac{1}{4}x + 1 \\ g(x) = \frac{1}{2}x - 2 \end{cases}$

20. $\begin{cases} y - x = 5 \\ 2x - 2y = 10 \end{cases}$

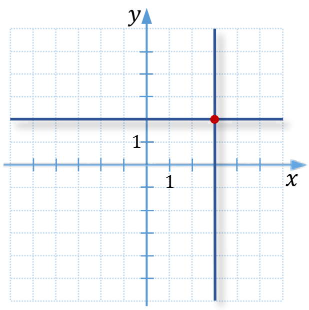
21. $\begin{cases} 6x - 2y = 2 \\ 9x - 3y = -1 \end{cases}$

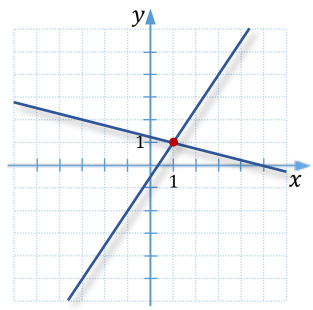
22. $\begin{cases} y = 3 - x \\ 2x + 2y = 6 \end{cases}$ **23.** $\begin{cases} 2x - 3y = 6 \\ 3y - 2x = -6 \end{cases}$ **24.** $\begin{cases} 2u + v = 3 \\ 2u = v + 7 \end{cases}$

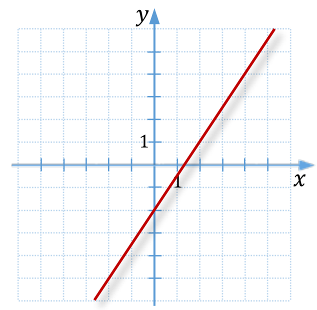
25. $\begin{cases} 2b = 6 - a \\ 3a - 2b = 6 \end{cases}$ **26.** $\begin{cases} f(x) = 2 \\ x = -3 \end{cases}$ **27.** $\begin{cases} f(x) = x \\ g(x) = -1.5 \end{cases}$

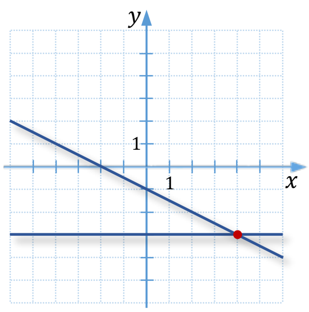
Concept Check

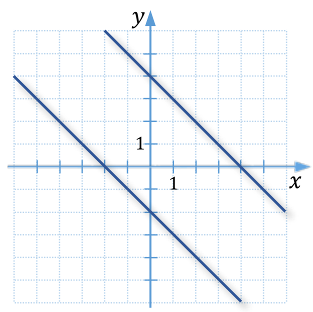
28. Classify each system **I** to **VI** as *consistent* or *inconsistent* and the equations as *dependent* or *independent*. Then, match it with the corresponding system of equations **A** to **F**.

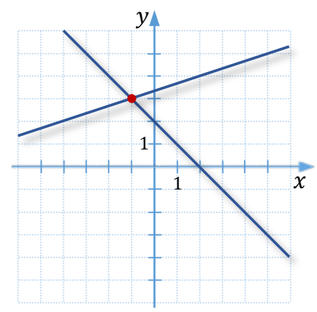
I


II


III


IV


V


VI


A $\begin{cases} 3y - x = 10 \\ x = -y + 2 \end{cases}$ **B** $\begin{cases} 9x - 6y = 12 \\ y = \frac{3}{2}x - 2 \end{cases}$ **C** $\begin{cases} 2y - 3x = -1 \\ x + 4y = 5 \end{cases}$

D $\begin{cases} x + y = 4 \\ y = -x - 2 \end{cases}$ **E** $\begin{cases} \frac{1}{2}x + y = -1 \\ y = -3 \end{cases}$ **F** $\begin{cases} x = 3 \\ y = 2 \end{cases}$

Solve each system by ***<u>substitution</u>****. If the system describes* ***parallel lines*** *or the* ***same line****, say so.*

29. $\begin{cases} y = 2x + 1 \\ 3x - 4y = 1 \end{cases}$ **30.** $\begin{cases} 5x - 6y = 23 \\ x = 6 - 3y \end{cases}$ **31.** $\begin{cases} x + 2y = 3 \\ 2x + y = 5 \end{cases}$

32. $\begin{cases} 2y = 1 - 4x \\ 2x + y = 0 \end{cases}$ **33.** $\begin{cases} y = 4 - 2x \\ y + 2x = 6 \end{cases}$ **34.** $\begin{cases} y - 2x = 3 \\ 4x - 2y = -6 \end{cases}$

35. $\begin{cases} 4s - 2t = 18 \\ 3s + 5t = 20 \end{cases}$ **36.** $\begin{cases} 4p + 2q = 8 \\ 5p - 7q = 1 \end{cases}$ **37.** $\begin{cases} \frac{x}{2} + \frac{y}{2} = 5 \\ \frac{3x}{2} - \frac{2y}{3} = 2 \end{cases}$

38. $\begin{cases} \frac{x}{4} + \frac{y}{3} = 0 \\ \frac{x}{8} - \frac{y}{6} = 2 \end{cases}$ **39.** $\begin{cases} 1.5a - 0.5b = 8.5 \\ 3a + 1.5b = 6 \end{cases}$ **40.** $\begin{cases} 0.3u - 2.4v = -2.1 \\ 0.04u + 0.03v = 0.7 \end{cases}$

*Solve each system by **elimination**. If the system describes **parallel lines** or the **same line**, say so.*

41. $\begin{cases} x+y=20 \\ x-y=4 \end{cases}$

42. $\begin{cases} 6x+5y=-7 \\ -6x-11y=1 \end{cases}$

43. $\begin{cases} x-y=5 \\ 3x+2y=10 \end{cases}$

44. $\begin{cases} x-4y=-3 \\ -3x+5y=2 \end{cases}$

45. $\begin{cases} 2x+3y=1 \\ 3x-5y=-8 \end{cases}$

46. $\begin{cases} -2x+5y=14 \\ 7x+6y=-2 \end{cases}$

47. $\begin{cases} 2x+3y=1 \\ 4x+6y=2 \end{cases}$

48. $\begin{cases} 6x-10y=-4 \\ 5y-3x=7 \end{cases}$

49. $\begin{cases} 0.3x-0.2y=4 \\ 0.2x+0.3y=1 \end{cases}$

50. $\begin{cases} \frac{2}{3}x+\frac{1}{7}y=-11 \\ \frac{1}{7}x-\frac{1}{3}y=-10 \end{cases}$

51. $\begin{cases} 3a+2b=3 \\ 9a-8b=-2 \end{cases}$

52. $\begin{cases} 5m-9n=7 \\ 7n-3m=-5 \end{cases}$

Discussion Point

53. A linear system of two equations has the solutions $(1,3)$ and $(4,-2)$. Are there any other solutions? If "no", explain why not. If "yes", find another solution.

Write each equation in slope-intercept form and then tell how many solutions the system has. <u>Do not</u> actually solve.

54. $\begin{cases} -x+2y=8 \\ 4x-8y=1 \end{cases}$

55. $\begin{cases} 6x=-9y+3 \\ 2x=-3y+1 \end{cases}$

56. $\begin{cases} y-x=6 \\ x+y=6 \end{cases}$

Solve each system by the method of your choice.

57. $\begin{cases} 3x+y=-7 \\ x-y=-5 \end{cases}$

58. $\begin{cases} 3x-2y=0 \\ 9x+8y=7 \end{cases}$

59. $\begin{cases} 3x-5y=7 \\ 2x+3y=30 \end{cases}$

60. $\begin{cases} 2x+3y=10 \\ -3x+y=18 \end{cases}$

61. $\begin{cases} \frac{1}{6}x+\frac{1}{3}y=8 \\ \frac{1}{4}x+\frac{1}{2}y=30 \end{cases}$

62. $\begin{cases} \frac{1}{2}x-\frac{1}{8}y=-\frac{1}{4} \\ 4x-y=-2 \end{cases}$

63. $\begin{cases} a+4b=2 \\ 5a-b=3 \end{cases}$

64. $\begin{cases} 3a-b=7 \\ 2a+2b=5 \end{cases}$

65. $\begin{cases} 6\cdot f(x)=2x \\ -7x+15\cdot g(x)=10 \end{cases}$

Analytic Skills

*Solve the system of linear equations. Assume that **a** and **b** represent nonzero constants.*

66. $\begin{cases} x+ay=1 \\ 2x+2ay=4 \end{cases}$

67. $\begin{cases} -ax+y=4 \\ ax+y=4 \end{cases}$

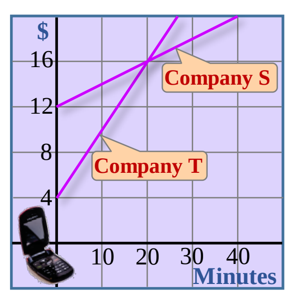
68. $\begin{cases} -ax+y=2 \\ ax+y=4 \end{cases}$

69. $\begin{cases} ax+by=2 \\ -ax+2by=1 \end{cases}$

70. $\begin{cases} 2ax-y=3 \\ y=5ax \end{cases}$

71. $\begin{cases} 3ax+2y=1 \\ -ax+y=2 \end{cases}$

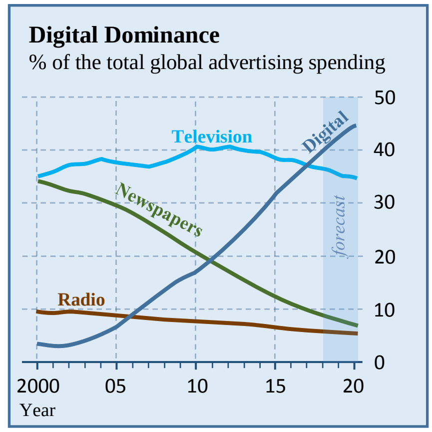
Refer to the accompanying graph to answer questions 63-64.



72. According to the graph, for how many air-time minutes do the two companies charge the same amount? Give an ordered pair of the form (minutes, dollars) to represent the situation.

73. For what range of air-time minutes is company T cheaper?

Refer to the accompanying graph to answer questions 65-68.



74. According to the predictions shown in the graph, what percent of the global advertising spending will be allocated to digital advertising in 2020? Give an ordered pair of the form (year, percent) to represent this information.

75. Estimate the year in which spending on digital advertising matches spending on television advertising.

76. Since when did spending on digital advertising exceed spending on advertising in newspapers? What was this spending as a percentage of global advertising spending at that time?

77. Since when did spending on digital advertising exceed spending on radio advertising? What was this spending as a percentage of global advertising spending at that time?

E.2 Applications of Systems of Linear Equations in Two Variables

Systems of equations are frequently used in solving applied problems. Although many problems with two unknowns can be solved with the use of a single equation with one variable, it is often easier to translate the information given in an application problem with two unknowns into two equations in two variables.

Here are some guidelines to follow when solving applied problems with two variables.

Solving Applied Problems with the Use of System of Equations

1. **Read** the problem, several times if necessary. When reading, watch the given information and what the problem asks for. Recognize the type of problem, such as geometry, total value, motion, solution, percent, investment, etc.
2. **Assign variables** for the unknown quantities. Use meaningful letters, if possible.
3. **Organize** the given information. Draw appropriate tables or diagrams; list relevant formulas.
4. **Write equations** by following a relevant formula(s) or a common sense pattern.
5. **Solve** the system of equations.
6. **Check** if the solution is reasonable in the context of the problem.
7. **State the answer** to the problem.

Below we show examples of several types of applied problems that can be solved with the aid of systems of equations.

Number Relations Problems

Example 1 ► **Finding Unknown Numbers**

Twice a number minus a second number equals 5. The sum of the two numbers is 16. Find the two numbers.

Solution ► Let a be the first number and b be the second number. The first sentence of the problem translates into the equation

$$2a - b = 5.$$

The second sentence translates to

$$a + b = 16.$$

Now, we can solve the system of the above equations, using the elimination method. Since

$$+\begin{cases} 2a - b = 5 \\ a + b = 16 \end{cases}$$

$$3a = 21 \qquad /\div 3$$

$$a = \mathbf{7},$$

then $b = 16 - a = 16 - 7 = \mathbf{9}$. Therefore, the two numbers are **7** and **9**.

Observation: *A single equation in two variables gives us infinitely many solutions. For example, some of the solutions of the equation* $a + b = 16$ *are* $(0, 16), (1, 15), (2, 14)$, *and so on. Generally, any ordered pair of the type* $(a, 16 - a)$ *is the solution to this equation. So, when working with two variables, to find a specific solution we are in need of a second equation (not equivalent to the first) that relates these variables. This is why problems with two unknowns are solved with the use of systems of two equations.*

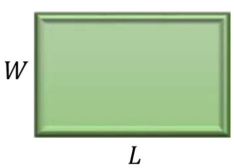
Geometry Problems

When working with geometry problems, we often use formulas for perimeter, area, or volume of basic figures. Sometimes, we rely on particular properties or theorems, such as *the sum of angles in a triangle is* $180°$ or *the ratios of corresponding sides of similar triangles are equal*.

Example 2 ► **Finding Dimensions of a Rectangle**

A homeowner sections off part of her backyard for a vegetable garden. The width of the garden is 4 ft shorter than the length. The perimeter of the garden is 52 ft. What are the dimensions of the garden?

Solution ► The problem refers to the perimeter of a rectangular garden. Suppose L and W represent the length and width of the rectangle. Then the perimeter is represented by the expression $2L + 2W$. Since the perimeter of the garden equals 52 feet, we set up the first equation



$$2L + 2W = 52 \qquad (1)$$

The second equation comes directly from translating the second sentence of the problem, which tells us that the width is 4 feet shorter than the length. So, we write

$$W = L - 4 \qquad (2)$$

Now, we can solve the system of the above equations, using the substitution method. After substituting equation (2) into equation (1), we obtain

$$2L + 2(L - 4) = 52$$

$$2L + 2L - 8 = 52 \qquad /+8$$

$$4L = 60 \qquad /\div 4$$

$$L = \mathbf{15}$$

So, $W = L - 4 = 15 - 4 = \mathbf{11}$.

Therefore, the garden is **15 feet** long and **11 feet** wide.

Number-Value Problems

Problems that refer to the number of different types of items and the value of these items are often solved by setting two equations. Either one equation compares the number of items, and the other compares the value of these items, like in coin types of problems, or both equations compare the values of different arrangements of these items.

Example 3 ► **Finding the Number of Each Type of Items**

Paint Town sold 45 paintbrushes, one kind at \$8.50 each and another at \$9.75 each. In all, \$398.75 was taken in for the brushes. How many of each type were sold?

Solution ► Let x represent the number of brushes at \$8.50 each, and let y represent the number of brushes at \$9.75 each. Then the value of x brushes at \$8.50 each is $8.50x$. Similarly, the value of y brushes at \$9.75 each is $9.75y$. To organize the given information, we suggest to create and complete the following table.

	brushes at $\$8.50$ each +	brushes at $\$9.75$ each	= Total
number of brushes	x	y	45
value of brushes (in \$)	$8.50x$	$9.75y$	398.75

Since we work with two variables, we need two different equations in these variables. The first equation comes from comparing the number of brushes, as in the middle row. The second equation comes from comparing the values of these brushes, as in the last row. So, we have the system

$$\begin{cases} x + y = 45 \\ 8.50x + 9.75y = 398.75 \end{cases}$$

to solve. This can be solved by substitution. From the first equation, we have $y = 45 - x$, which after substituting to the second equation gives us

$$8.50x + 9.75(45 - x) = 398.75 \qquad /\cdot 100$$

$$850x + 975(45 - x) = 39875$$

$$850x + 43875 - 975x = 39875 \qquad /-43875$$

$$-125x = -4000 \qquad /\div(-125)$$

$$x = \mathbf{32}$$

Then, $y = 45 - x = 45 - 32 = \mathbf{13}$.

Therefore, there were **32** brushes sold at \$8.50 each and **13** brushes sold at \$9.75 each.

Example 4 ► **Finding the Unit Cost of Each Type of Items**

A carpenter purchased 60 ft of redwood and 80 ft of pine for a total cost of \$286. A second purchase, at the same prices, included 100 ft of redwood and 60 ft of pine for a total cost of \$396. Find the cost per foot of redwood and of pine.

Solution ► Let r be the cost per foot of redwood and let p be the cost per foot of pine. Then the value of 60 ft of redwood is represented by $60r$, and the value of 80 ft of pine is represented by $80p$. Using the total cost of \$286, we write the first equation

$$60r + 80p = 286$$

Similarly, using the total cost of \$396 for 100 ft of redwood and 60 ft of pine, we write the second equation

$$100r + 60p = 396$$

We will solve the system of the above equations via the elimination method.

$$\begin{cases} 60r + 80p = 286 \\ 100r + 60p = 396 \end{cases} \qquad \begin{array}{l} /\cdot(-3) \\ /\cdot 4 \end{array}$$

$$+\begin{cases} -180r - 240p = -858 \\ 400r + 240p = 1584 \end{cases}$$

$$220r = 726 \qquad /\div 220$$

$$r = \mathbf{3.30}$$

After substituting $r = 3.30$ into the first equation, we obtain

$$60 \cdot 3.30 + 80p = 286$$

$$198 + 80p = 286$$

$$80p = 88$$

$$p = \mathbf{1.10}$$

So, redwood costs **3.30 \$/ft**, and pine costs **1.10 \$/ft**.

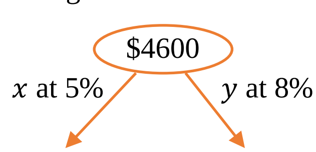
Investment Problems

Investment problems usually involve calculation of simple annual interest according to the formula $\boldsymbol{I = Prt}$, where I represents the amount of interest, P represents the principal which is the amount of investment, r represents the annual interest rate in decimal form, and t represents the time in years.

Example 5 ► **Finding the Amount of Each Loan**

A student takes out two loans totaling \$4600 to help pay for college expenses. One loan is at 5% annual interest, and the other is at 8% annual interest. The first-year interest is \$278. Find the amount of each loan.

Solution ► Suppose the amount of loan taken at 5% is x, and the amount of loan taken at 8% is y. The situation can be visualized by the diagram



one year interest: $0.05x + 0.08y = 278$

So, the system of equations to solve is

$$\begin{cases} x + y = 4600 \\ 0.05x + 0.08y = 278 \end{cases}$$

Using the substitution method, we solve the first equation for x,

$$x = 4600 - y, \qquad (*)$$

substitute it into the second equation,

decimal elimination is optional

$$0.05(4600 - y) + 0.08y = 278, \qquad /\cdot 100$$

and after elimination of decimals via multiplication by 100,

$$5(4600 - y) + 8y = 27800,$$

solve it for y:

$$23000 - 5y + 8y = 27800 \qquad /-23000$$

$$3y = 4800$$

$$y = \mathbf{1600}$$

Then, after plugging in the y-value to the substitution equation $(*)$, we obtain

$$x = 4600 - 1600 = \mathbf{3000}$$

So, the amount of loan taken at 5% is \$**3000**, and the amount of loan taken at 8% is \$**1600**.

Mixture – Solution Problems

In mixture or solution problems, we typically mix two or more mixtures or solutions with different concentrations of a particular substance that we will refer to as the content. For example, if we are interested in the salt concentration in salty water, the salt is referred to as the content. When solving mixture problems, it is helpful to organize data in a table such as the one shown below.

	% ·	volume =	content
type I			
type II			
mixture/solution			

Example 6 ► **Solving a Mixture Problem**

The owner of a natural foods store made 20 lb of a dried fruit and granola blend that is 50% dried fruit by mixing a dried fruit and granola blend that is 20% dried fruit with a blend that is 60% dried fruit. How many pounds of each type were used to get the desired blend?

Solution ► Suppose x is the amount of the 20% blend and y is the amount of the 60% blend. We complete the table

	% ·	volume =	dried fruit
20% blend	0.2	x	$0.2x$
60% blend	0.6	y	$0.6y$
50% blend	0.5	20	10

The last column is completed by multiplying the "%" and "volume" columns.

The first equation comes from combining the weight of blends as indicated in the "volume" column. The second equation comes from combining the weight of dried fruit, as indicated in the last column. So, we solve

$$\begin{cases} x + y = 20 \\ 0.2x + 0.6y = 10 \end{cases}$$

Using the substitution method, we solve the first equation for x,

$$x = 20 - y, \qquad (*)$$

and substitute it into the second equation and solve it for y:

$$0.2(20 - y) + 0.6y = 10 \qquad /\cdot 10$$

$$2(20 - y) + 6y = 100$$

$$40 - 2y + 6y = 100 \qquad /-40$$

$$4y = 60 \qquad /\div 4$$

$$y = \mathbf{15}$$

Then, using the substitution equation $(*)$, we find the value of x:

$$x = 20 - 15 = \mathbf{5}$$

So, to obtain the desired blend, **5 lb** of 20% and **15 lb** of 60% blend was used.

Motion Problems

In motion problems, we follow the formula $\boldsymbol{Rate \cdot Time = Distance}$. Drawing a diagram and completing a table based on the $\boldsymbol{R \cdot T = D}$ formula is usually helpful. In some motion problems, in addition to the rate of the moving object itself, we need to consider the rate of a moving medium such as water current or wind. The overall rate of a moving object is typically either the sum or the difference between the object's own rate and the rate of the moving medium.

Example 7 ► **Finding Rates in a Motion Problem**

A car travels 250 km in the same time that a truck travels 225 km. If the rate of the car is 8 kilometers per hour faster than the rate of the truck, find both rates.

Solution ► Using meaningful letters, let c and t represent the rate of the car and the truck, respectively. Since the rate of the car, c, is 8 kph faster than the rate of the truck, t, we can write the first equation:

$$c = t + 8$$

The second equation comes from comparing the travel time of each vehicle, as indicated in the table below.

	R ·	T	$= D$
car	c	$\frac{250}{c}$	250
truck	t	$\frac{225}{t}$	225

To find the expression for time, we follow the formula $T = \frac{D}{R}$, which comes from solving $R \cdot T = D$ for T.

So, we need to solve the system

$$\begin{cases} c = t + 8 \\ \frac{250}{c} = \frac{225}{t} \end{cases}$$

Cross-multiplication can **only** be applied to a **proportion** (an equation with a single fraction on each side.)

Notice that multiplication by the **LCD** would give the same result.

After substituting the first equation into the second, we obtain

$$\frac{250}{t+8} = \frac{225}{t},$$

which can be solved by cross-multiplying

$$250t = 225(t+8)$$

$$250t = 225t + 1800 \qquad /-225t$$

$$25t = 1800 \qquad /\div 25$$

$$t = \mathbf{72}$$

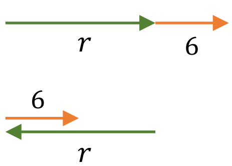
Then, we use this value to find $c = t + 8 = 72 - 8 = \mathbf{64}$.

So, the rate of the car is **64 kph**, and the rate of the truck is **72 kph**.

Example 8 ► **Solving a Motion Problem with a Current**

Mia's motorboat took 3 hr to make a trip downstream with a 6-mph current. The return trip against the same current took 5 hr. Find the speed of the boat in still water.

Solution ►



Let r be the speed of the boat in still water.
Then the speed of the boat moving downstream is 6 mph faster because of the current going in the same direction as the boat. So, it is represented by $r + 6$.
The speed of the boat moving against the current is 6 mph slower. So, it is represented by $r - 6$.
Also, let d represent the distance covered by the boat going in one direction.

To organize the information, we can complete the table below.

	R ·	T	$= D$
downstream	$r + 6$	3	d
upstream	$r - 6$	5	d

The two equations come from following the formula $R \cdot T = D$, as indicated in each row.

$$\begin{cases}(r+6)\cdot 3 = d\\(r-6)\cdot 5 = d\end{cases}$$

Since left sides of both equations represent the same distance d, we can equal them and solve for r:

$$(r+6)\cdot 3 = (r-6)\cdot 5$$
$$3r + 18 = 5r - 30 \qquad /-3r, +30$$
$$48 = 2r \qquad /\div 2$$
$$\mathbf{24} = r$$

So, the speed of the boat in still water is **24 mph**.

E.2 Exercises

Concept Check

1. If a container of liquid contains 60 liters of 40% acid solution, what is the volume of pure acid?

2. If \$5000 is invested in an account paying 3% simple annual interest, how much interest will be earned during the first year?

3. If 1 kilogram of turkey costs \$8.49, give an expression for the cost of x kilograms.

4. If 1 ticket to the movie Avatar costs \$9 and y tickets are sold, give an expression for the amount collected from the sale.

5. If the speed of a motorboat in still water is x kph and the speed of a current is c kph, give an expression for the speed of this boat travelling **a.** downstream, **b.** upstream.

6. If a plane travels with its own rate of r kph and there is a wind blowing at w kph, give an expression for the rate of this plane travelling with the **a.** headwind, **b.** tailwind.

Solve each problem using two variables.

7. Two angles are complementary, which means that their sum is 90°. The measure of the larger angle is 9° more than eight times the measure of the smaller angle. Find the measures of the two angles.

8. Two angles are supplementary, which means that their sum is 180°. The measure of the larger angle is 40° more than three times the measure of the smaller angle. Find the measures of the two angles.

9. The sum of the base and the height of a triangle is 192 inches. The height is twice the base. Find the base and the height.

10. The width of a standard tennis court used for playing doubles is 42 ft less than the length. The perimeter of the court is 228 ft. Find the dimensions of the court.

11. A marathon is a run that covers about 26 mi. Paula Radcliffe of Great Britain holds the world record time in the women's marathon. At one point in her record-setting race, she was three times as far from the end of the course as she was from the starting point. How many miles had she run at that time?

12. Jane asked her students to determine the two numbers that she placed in a sealed envelope. The smaller number is 2 more than one-third of the larger number. Three times the larger number is 1 less than eight times the smaller. Find the numbers.

13. In the 2008 Summer Olympics in Beijing, China, Russia earned 5 fewer gold medals than bronze. The number of silver medals earned was 35 less than twice the number of bronze medals. Russia earned a total of 72 medals. How many of each kind of medal did Russia earn?

14. A piece of wire that is 102 cm long is to be cut into two pieces, each to be bent to make a square. The length of a side of one square is to be twice the length of a side of the other. How should the wire be cut?

15. A coffee shop sells 12-oz and 20-oz cups of coffee. On a particular day, the shop sold a total of 508 cups of coffee. If the shop sold 3 times as many 20-oz cups of coffee as 12-oz cups of coffee, how many cups of each size did the coffee shop sell that day?

16. Mountainside Fleece sold 40 neckwarmers. Solid-color neckwarmers sold for $9.90 each and print ones sold for $12.75 each. In all, $421.65 was taken in for the neckwarmers. How many of each type were sold?

17. A local performing-arts group holds its annual winter arts festival in December. The group charges $20 for adult admission tickets and $12 for children's admission tickets. The group raised $13,344 on the sale of 824 tickets. How many of each type of ticket were sold?

18. The total of Megan's writing and math scores on the SAT was 1094. Her math score was 248 points higher than her writing score. What were her math and writing SAT scores?

19. Andrew McGinnis works at Arby's. During one particular day, he sold 15 Junior Roast Beef sandwiches and 10 Big Montana sandwiches, totaling $75.25. Another day he sold 30 Junior Roast Beef sandwiches and 5 Big Montana sandwiches, totaling $84.65. How much did each type of sandwich cost?

20. At a business meeting at Panera Bread, the bill for two cappuccinos and three house lattes was $14.55. At another table, the bill for one cappuccino and two house lattes was $8.77. How much did each type of beverage cost?

21. New York City and Washington, D.C., were the two most expensive cities for business travel in 2009. Based on the average total costs per day for each city (which include a hotel room, car rental, and three meals), 2 days in New York and 3 days in Washington cost $2772, while 4 days in New York and 2 days in Washington cost $3488. What was the average cost per day in each city?

22. A public storage facility rents small and large storage lockers. A small storage locker has 200 ft^2 of storage space, and a large storage locker has 800 sq ft of storage space. The facility has 54 storage lockers and a total of 22,800 ft^2 of storage space. How many large storage lockers does the facility have?

23. Brandt's student loans totaled $3500. Part of this loan was borrowed at 3% interest and the rest at 2.5%. After one year, Brandt had accumulated $97.50 in interest. What was the amount of each loan?

24. An executive nearing retirement made two investments totaling $45,000. In one year, these investments yielded $2430 in simple interest. Part of the money was invested at 4% and the rest at 6%. How much was invested at each rate?

25. Twice the amount of money that was invested in a low-risk fund was invested in a high-risk fund. After one year, the low-risk fund earned 6%, and the high-risk fund lost 9%. The investments had a net loss of $210. How much was invested in each fund?

26. Mary has two accounts to choose from, one at 5.5% and the other at 7%. Mary invested $2,000 more in the 7% account than the 5.5% account. If the total interest for one year was $827.50, then how much was invested in each account?

27. How many ounces of 5% hydrochloric acid, 20% hydrochloric acid, and water must be combined to get 10 oz of solution that is 8.5% hydrochloric acid if the amount of water used must equal the total amount of the other two solutions?

28. A sample of dimes and quarters totals $18. If there are 111 coins in all, how many of each type of coin are there?

29. A pharmacist has two vitamin-supplement powders. The first powder is 20% vitamin B1 and 10% vitamin B2. The second is 15% vitamin B1 and 20% vitamin B2. How many milligrams of each powder should the pharmacist use to make a mixture that contains 130 mg of vitamin B1 and 80 mg of vitamin B2?

30. Soybean meal is 16% protein, and cornmeal is 9% protein. How many pounds of each should be mixed to get a 350-kg mixture that is 12% protein?

31. Christina makes a $9.25 purchase at a bookstore in Reno with a $20 bill. The store has no bills and gives her the change in quarters and dollar coins. There are 19 coins in all. How many of each kind are there?

32. A motorboat traveling with the current went 36 km in 2 h. Against the current, it took 3 h to travel the same distance. Find the rate of the boat in the calm water and the rate of the current.

33. A cabin cruiser traveling with the current went 45 km in 3 h. Against the current, it took 5 h to travel the same distance. Find the rate of the cabin cruiser in calm water and the rate of the current.

34. A jet plane flying with the wind went 2200 km in 4 h. Against the wind, the plane could fly only 1840 km in the same amount of time. Find the rate of the plane in the calm air and the rate of the wind.

35. Flying with the wind, a pilot flew 1200 km between two cities in 4 h. The return trip against the wind took 5 h. Find the rate of the plane in the calm air and the rate of the wind.

36. A rowing team rowing with the current traveled 20 km in 2 h. Rowing against the current, the team rowed 12 km in the same amount of time. Find the rate of the team in calm water and the rate of the current.

37. A plane flying with a tailwind flew 1500 km in 5 h. Against the wind, the plane required 6 h to fly the same distance. Find the rate of the plane in the calm air and the rate of the wind.

Analytic Skills *Solve each problem using two variables.*

38. Rod is a pilot for Crossland Airways. He computes his flight time against a headwind for a trip of 2900 mi at 5 hr. The flight would take 4 hr and 50 min if the headwind were half as great. Find the headwind and the plane's airspeed (speed in still air).

39. Donna is late for a sales meeting after traveling from one town to another at a speed of 32 mph. If she had traveled 4 mph faster, she could have made the trip in 1 hr less time. How far apart are the towns?

40. A truck and a car leave a service station at the same time and travel in the same direction. The truck travels at 96 kph and the car at 108 kph. They can maintain CB radio contact within a range of 10 km. When will they lose contact?

41. The radiator in Michelle's car contains 16 L of antifreeze and water. This mixture is 25% antifreeze. How much of this mixture should she drain and replace with pure antifreeze so that there will be a mixture of 50% antifreeze?

42. Bennet Custom Flooring has 0.5 gallons of stain that is 20% brown and 80% neutral. A customer orders 1.5 gallons of a stain that is 60% brown and 40% neutral. How much of the pure brown stain and how much of the neutral stain should be added to the original 0.5 gallons to make up the order?