2.1, 2.2   Linear Equations in One Variable and Formulas

expression – sequence of numbers, variables, operation symbols and grouping symbols
equation – two expressions connected by an equal sign
equations can be solved, while expressions can only be simplified or evaluated

ex. $\frac{1}{2}(2x + 6) = 0$ $\frac{1}{2}(2x + 6)$
            equation to solve   **expression** to simpify or evaluate

linear equation in one variable -   an equation that can involve a variable but only in the first power; such equation can be rewritten in the following, equivalent form $Ax + B = 0$

ex. $2x - 7 = 3$ $5x - 1 = 5x + 4$ $1 = 1$

ex. of nonlinear equations: $\frac{8}{x} = 5$ $\sqrt{x} + 3 = 7$ $x^2 + 3x = -2$

solution – a value of the variable that will make the equation true
solution set – a set of all possible solutions

Example 1: Find the solution set for the following equations:
a) $2x - 7 = 3$ b) $5x - 1 = 5x + 4$ c) $1 = 1$

If a linear equation has   a) one solution, it is called a **conditional** equation
                            b) infinitely many solutions (all real numbers), it is an **identity**
                            c) no solutions, it is a **contradiction**

equivalent equations – equations with the same solution sets

We can produce equivalent equations by
    -    adding to both sides of the equation the same value,
    -    multiplying both sides of the equation by the same, nonzero value

Practice:
1.   Solve.
a) $4[2x - (3 - x) + 5] = -(2 + 7x)$ b) $\frac{2x+5}{5} = \frac{3x+1}{2} + \frac{-x+7}{2}$

General Guidelines for Solving Linear Equations:

• **Clear fractions or decimals** by multiplying both sides of the equation by LCD, or a power of 10 • **Open brackets** by applying the distributive property • **Collect and combine** variable terms on one side of the equation and free numbers on the other side • **Isolate the variable** by dividing by the linear coefficient	$\frac{1}{8}(2x - 1) + 3 = \frac{5}{6}(5 - 3x)$ $\quad$ $/\cdot 24$ $3(2x - 1) + 72 = 20(5 - 3x)$ $6x - 3 + 72 = 100 - 60x$ $6x + 60x = 100 + 3 - 72$ $66x = 31$ $\qquad$ $/\div 66$ $x = \dfrac{31}{66}$

Solving Formulas:
formula – a general rule describing relationship between various quantities (variables)

When solving formulas for a given variable, remember to
- highlight the variable of interest and treat other variables as numbers
  ex. $2L + 2W = P$ solve for L

- think about undoing (reversing) operations to reach the given variable;
  here are the pairs of reversing operations:
  $$\begin{array}{cc} + & - \\ \cdot & \div \\ (*)^2 & \sqrt{*} \end{array}$$

  ex. $\dfrac{h(a+b)}{2} = A$ solve for a

- keep the given variable in the numerator
  ex. $r = \dfrac{d}{t}$ solve for t
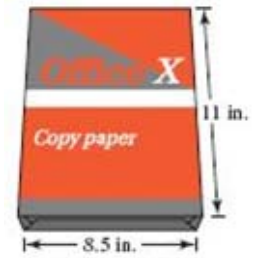
- keep the given variable in one place
  ex. $\pi r + 3r = P$ solve for r

2.   A sheet of standard-size copy paper has the width W = 8.5 in and the length L= 11 in. A ream of this paper has 500 sheets and a volume V = 187 in³.
If x represents the thickness of one sheet,
a)   write a formula for the volume V of the ream in terms of the width W, length L, and thickness x,

b)   solve this formula for x,

c)   and evaluate it to find the thickness of one such sheet of paper.