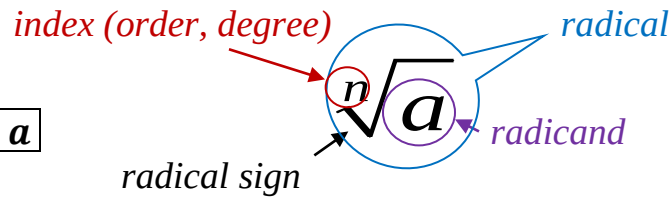
8.1 Radical Expressions, Functions, and Graphs

Operation of taking ***n*-th degree root** is a reverse process to raising to a power of ***n***.



Definition: $\boxed{\sqrt[n]{a} = b \;\Leftrightarrow\; b^n = a}$

Notice! If ***n*** is **even**, the above definition allows for dual answer. For example if $n = 2$ and $a = 4$, then according to the definition, b could be 2 or -2.
We would like to have a uniquely defined answer, so we agree to take the **positive answer** only. This is called the **principal root**.
So $\sqrt[even]{positive} = positive$; also notice that $\sqrt[even]{negative} = DNE$

We don't have such problems when ***n*** is **odd**.
$\sqrt[odd]{positive} = positive$ and $\sqrt[odd]{negative} = negative$

Example 1: Simplify radicals.

a) $\sqrt[4]{16} =$

b) $\sqrt[5]{-32} =$

c) $\sqrt[3]{\frac{8}{27}} =$

d) $\sqrt{\frac{.0625}{.49}} =$

e) $\sqrt[8]{5^8} =$

f) $\sqrt[2011]{5^{2011}} =$

g) $\sqrt[10]{(-3)^{10}} =$

h) $\sqrt[9]{(-3)^9} =$

Notice!	If ***n*** is **even**,	$\sqrt[n]{x^n} = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases} = \lvert x \rvert$
	If ***n*** is **odd**,	$\sqrt[n]{x^n} = x$

Example 2: Simplify radicals.

a) $\sqrt{a^2} =$

b) $\sqrt{x^6} =$

c) $\sqrt[4]{(-x)^4} =$

d) $\sqrt[3]{y^3} =$

e) $-\sqrt[8]{(-x)^8} =$

f) $-\sqrt[5]{-x^5} =$

Example 3: Using a calculator, approximate each radical to 2 decimal places.

a) $\sqrt{2} \approx$ b) $\sqrt{3} \approx$ c) $\sqrt[3]{10} \approx$ d) $\sqrt[5]{100} \approx$

Example 4: Estimate each root to the nearest whole number <u>without</u> a calculator.

a) $\sqrt{17} \approx$ b) $\sqrt{70} \approx$ c) $\sqrt{115} \approx$ d) $\sqrt{200} \approx$

Heron's Formula (to find the area of a triangle, knowing the lengths of its sides)

Suppose that a, b, and c are the lengths of the sides of a triangle.
Let $\boldsymbol{s = \frac{1}{2}(a + b + c)}$ be its ***semiperimeter***.
Then

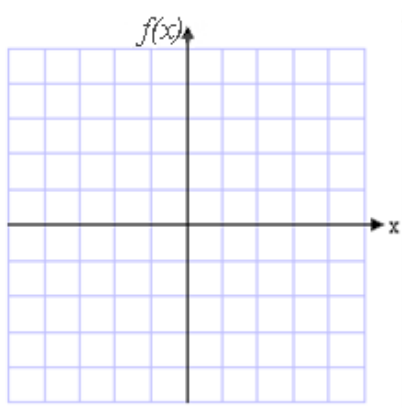
$$\boldsymbol{\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}}$$

represents the area of this triangle.

Example 5:
Find the area of the Bermuda Triangle, to the nearest thousand square miles, if the "sides" of this triangle measure approximately 850 mi, 925 mi, and 1300 mi.

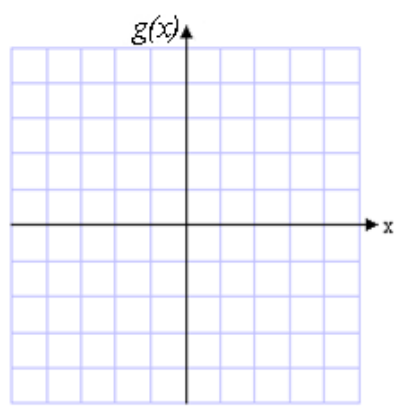
Example 6: Graph each function and state its domain and range.

a) $f(x) = \sqrt{x}$



Domain: .....
Range: .....

b) $g(x) = \sqrt[3]{x}$



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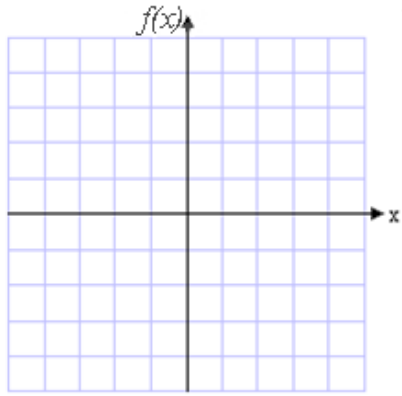
Observe transformations of the basic shapes when graphing the following functions:

c) $f(x) = \sqrt{x-1}$



Domain: .....

Range: .....

Transformation:

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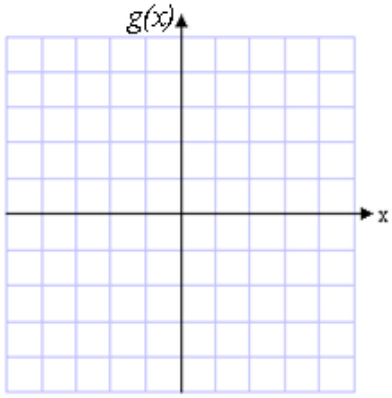
d) $g(x) = -\sqrt[3]{x} + 2$



Domain: .....

Range: .....

Transformation:

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