Additional Functions, Conic Sections, and Nonlinear Systems

Relations and functions are an essential part of mathematics as they allow to describe interactions between two or more variable quantities. In this chapter, we will give a quick overview of the commonly used functions, their properties such as domain and range, and some of their transformations, particularly translations. Some of these functions (i.e., linear, quadratic, square root, and reciprocal functions) were already discussed in detail in *sections G1*, *Q3*, *RD1*, and *RT5*. In *section C1*, we will explore some additional functions, such as absolute value or greatest integer functions, as well as functions of the form $\frac{1}{f(x)}$ or $|f(x)|$.

Aside from new functions, we will discuss equations and graphs of commonly used relations such as circles, ellipses, and hyperbolas. These relations are known as conic sections as their graphs have the shape of a curve formed by the intersection of a cone and a plane. Conic sections are geometric representations of quadratic equations in two variables and as such, they include parabolas. Thus, studying conic sections is an extension of studying parabolas. When working with conic sections, we are often in need of finding intersection points of given curves. Thus, at the end of this chapter, we will discuss solving systems of nonlinear equations as well as nonlinear inequalities.

C.1 Properties and Graphs of Additional Functions

The graphs of some **basic functions**, such as

$$f(x) = x^2,\ \ f(x) = |x|,\ f(x) = \sqrt{x}, \text{ or } f(x) = \frac{1}{x},$$

were already presented throughout this text. Knowing the shapes of the graphs of these functions is very useful for graphing related functions, such as $g(x) = |x| - 2$ or $f(x) = \sqrt{x+1}$. In *section Q3*, we observed that the graph of function $g(x) = (x-p)^2 + q$ could be obtained by translating a graph of the basic parabola p units horizontally and q units vertically. This observation applies to any function $f(x)$.

To graph a function $\boldsymbol{f(x-a)+b}$, it is enough to translate the graph of $\boldsymbol{f(x)}$ by ***a* units horizontally** and ***b* units vertically**.

Examine the relations between the defining formula of a function and its graph in the following examples.

Basic Functions and Their Translations

Parabola $f(x) = x^2$

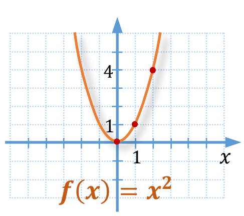


Figure 1.1a

Recall the shape of the graph of the basic parabola $f(x) = x^2$, as in *Figure 1.1a*. The domain of this function is $\mathbb{R}$, and the range is the interval $[0, \infty)$.

The graph of the basic parabola can be used to graph other quadratic functions such as $\boldsymbol{g(x) = (x-3)^2 - 1}$. Function g

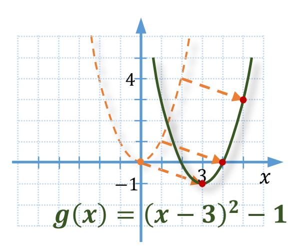


Figure 1.1b

can be graphed by translating the basic parabola **3 units to the right** and **1 unit down**, as in *Figure 1.1b*.

Observe that under this translation,

- the vertex $(0,0)$ of the basic parabola is moved to the **vertex** $(\mathbf{3},\mathbf{-1})$ of function g;
- the **domain** of function g remains unchanged, and it is still $\mathbb{R}$;
- the **range** of function g is the interval $[\mathbf{-1},\infty)$ as a result of the translation of the range $[0,\infty)$ of the basic parabola by 1 unit down.

Absolute Value $f(x) = |x|$

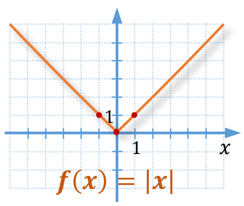


Figure 1.2a

x	$f(x)$
−2	2
−1	1
0	0
1	1
2	2

Using a table of values, we can graph the basic absolute value function, $f(x) = |x|$, as in *Figure 1.2a*. The domain of this function is $\mathbb{R}$, and the range is the interval $[0,\infty)$. Similarly as in the case of the basic parabola, the lowest point, called the vertex, is at $(0,0)$.

The graph of the basic absolute value function can be used to graph other absolute value functions such as $\boldsymbol{g(x) = |x+1| + 2}$. Function g can be graphed by translating function $f(x) = |x|$ by **1 unit to the left** and **2 units up**, as in *Figure 1.2b*.

x	$g(x)$
−3	4
−2	3
−1	2
0	3
1	4

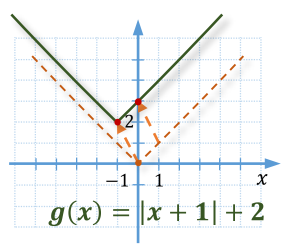


Figure 1.2b

Observe that under this translation,

- the vertex $(0,0)$ of function f is moved to the **vertex** $(\mathbf{-1},\mathbf{2})$ of function g;
- the **domain** of function g remains unchanged and it is still $\mathbb{R}$;
- the **range** of function g is the interval $[\mathbf{2},\infty)$, as a result of the translation of the range $[0,\infty)$ of function f by 2 units up.

Square Root $f(x) = \sqrt{x}$

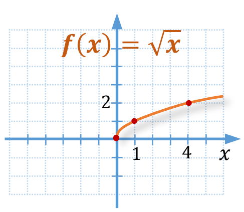


Figure 1.3a

x	$f(x)$
−2	2
−1	1
0	0
1	1
2	2

Using a table of values, we can graph the basic square root function, $f(x) = \sqrt{x}$, as in *Figure 1.3a*. The domain of this function is the interval $[0,\infty)$, and the range is also $[0,\infty)$. The curve starts at the origin $(0,0)$.

The graph of the basic square root function can be used to graph other square root functions such as $\boldsymbol{g(x) = \sqrt{x+1} - 2}$. Function g can be graphed by translating function $f(x) = \sqrt{x}$ by **1 unit to the left** and **2 units down**, as in *Figure 1.3b*.

x	$g(x)$
−2	2
−1	1
0	0
1	1
2	2

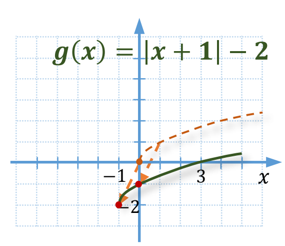


Figure 1.3b

Observe that under this translation,

- the initial point $(0,0)$ of function f is moved to the **initial point** $(\mathbf{-1},\mathbf{2})$ of function g;
- the **domain** of function g is moved to $[\mathbf{-1},\infty)$, by subtracting 1 from all domain values $[0,\infty)$ of function f;
- the **range** of function g is moved to $[\mathbf{-2},\infty)$, by subtracting 2 from all range values $[0,\infty)$ of function f.

Cubic $f(x) = x^3$

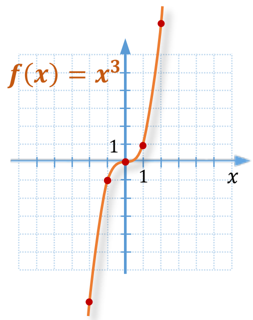


Figure 1.4a

x	$f(x)$
-2	-8
-1	-1
$-\frac{1}{2}$	$-\frac{1}{8}$
0	1
$\frac{1}{2}$	$\frac{1}{8}$
1	1
2	8

x	$g(x)$
-2	2
-1	1
0	0
1	1
2	2

Using a table of values, we can graph the basic cubic function, $f(x) = x^3$, as in *Figure 1.4a*. The domain and range of this function are both $\mathbb{R}$. The curve is symmetric about the origin $(0,0)$.

The graph of the basic cubic function can be used to graph other cubic functions such as $\boldsymbol{g(x) = (x-3)^3 - 2}$. Function g can be graphed by translating function $f(x) = x^3$ by **3 units to the right** and **2 units down**, as in *Figure 1.4b*.

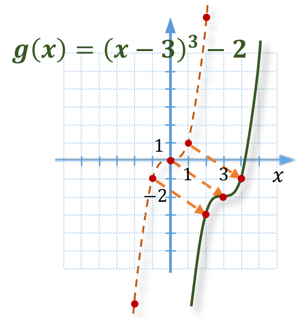


Figure 1.4b

Observe that under this translation,

- the central point $(0,0)$ of function f is moved to the **central point** $(\mathbf{3}, \mathbf{-2})$ of function g;
- the **domain** of function g remains unchanged, and it is still $\mathbb{R}$;
- the **range** of function g remains unchanged, and it is still $\mathbb{R}$.

Reciprocal $f(x) = \frac{1}{x}$

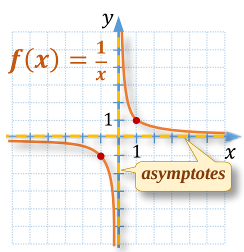


Figure 1.5a

x	$f(x)$
-4	$-\frac{1}{4}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	*undefined*
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
4	$\frac{1}{4}$

x	$g(x)$
0	$\frac{1}{2}$
1	0
$\frac{3}{2}$	-1
2	*undefined*
$\frac{5}{2}$	3
3	2
4	$\frac{3}{2}$

Using a table of values, we can graph the basic reciprocal function, $f(x) = \frac{1}{x}$, as in *Figure 1.5a*. The domain and range of this function is the set of all real numbers except for zero, $\mathbb{R} \setminus \{\mathbf{0}\}$. The graph consists of two curves that are approaching two asymptotes, the horizontal asymptote $y = 0$ and the vertical asymptote $x = 0$.

The graph of the basic reciprocal function can be used to graph other reciprocal functions such as $g(x) = \frac{1}{x-2} + 1$. Function g can be graphed by translating function $f(x) = \frac{1}{x}$ by 2 units to the right and 1 unit up, as in *Figure 1.5b*.

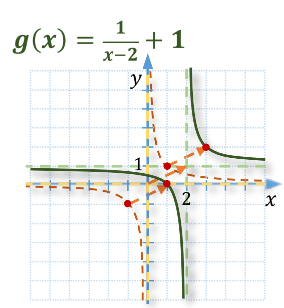


Figure 1.5b

Observe that under this translation,

- the **horizontal asymptote** of function f is moved **1 unit up**, and the **vertical asymptote** of function f is moved **2 units to the right**;
- the **domain** of function g is moved to $\mathbb{R} \setminus \{\mathbf{2}\}$, by adding 2 to all domain values $\mathbb{R} \setminus \{0\}$ of function f;
- the **range** of function g is moved to $\mathbb{R} \setminus \{\mathbf{1}\}$, by adding 1 to all range values $\mathbb{R} \setminus \{0\}$ of function f.

Greatest Integer $f(x) = [\![x]\!]$

Definition 1.1 ► The greatest integer, denoted $[\![x]\!]$, of a real number x is the **greatest integer that does not exceed this number** x. For example,

$$[\![0.9]\!] = 0, \quad [\![1]\!] = 1, \quad [\![1.1]\!] = 1, \quad [\![1.9]\!] = 1$$

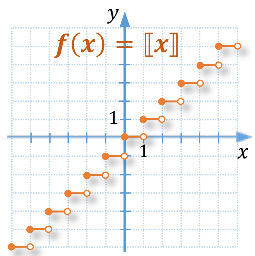


Figure 1.6a

x	$f(x)$
-1.5	-2
-1.1	-2
-1	-1
-0.5	-1
-0.1	-1
0	0
0.5	0
0.9	0
1	1
1.2	1
2	2

x	$g(x)$
-0.5	1
0	2
0.5	2
1	3
1.5	3

Using a table of values, we can graph the basic greatest integer function, $f(x) = [\![x]\!]$, as in *Figure 1.6a*. The domain of this function is the set of real numbers $\mathbb{R}$ while the range is the set of integers $\mathbb{Z}$. The graph consists of infinitely many half-open segments that line up along the diagonal, $y = x$.

The graph of the basic greatest integer function can be used to graph other greatest integer functions such as $\boldsymbol{g(x) = [\![x]\!] + 2}$. Function g can be graphed by translating function $f(x) = [\![x]\!]$ by 1 unit up, as in *Figure 1.6b*.

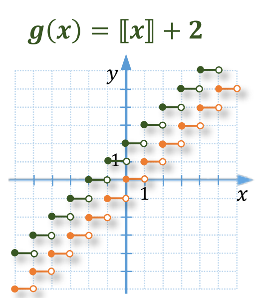


Figure 1.6b

Observe that under this translation,

- the segments of the graph g line up along the line $y = x + 2$;
- the **domain** of function g remains unchanged, and it is still $\mathbb{R}$;
- the **range** of function g remains unchanged, and it is still the set of all integers $\mathbb{Z}$;

Other Transformations of Basic Functions

Aside from translating, graphs can be transformed by flipping them along x- or y-axis, or stretching or shrinking (dilating) in different directions. In the next two examples, observe the relation between the defining formula of a function and the graph transformation of the corresponding basic function.

Example 1 ► **Graphing Functions and Identifying Transformations**

Graph each function. State the transformation(s) of the corresponding basic function that would result in the obtained graph. Then, describe the main properties of the function, such as domain, range, vertex, asymptotes, and symmetry, if applicable.

a. $f(x) = -\frac{1}{x}$ **b.** $f(x) = 2[\![x]\!]$ **d.** $f(x) = \frac{1}{2}|x| - 3$

Solution ► **a.** To graph $f(x) = -\frac{1}{x}$, we first observe that this is a modified reciprocal function. So, we expect that the graph might have some asymptotes.

Since we cannot divide by zero, $x = 0$ does not belong to the domain of this function. This suggests that the graph may have a vertical asymptote, $x = 0$. Also, since the numerator of the fraction $-\frac{1}{x}$ is never equal to zero, then function $f(x) = -\frac{1}{x}$ would never assume the value of zero. So, zero is out of the range of this function. This suggests that the graph may have a horizontal asymptote, $y = 0$.

After graphing the two asymptotes and plotting a few points of the graph, we obtain the final graph, as in *Figure 1.7*.

x	$f(x)$
-2	$\frac{1}{2}$
-1	1
$-\frac{1}{2}$	2
0	*undefined*
$\frac{1}{2}$	-2
1	-1
2	$-\frac{1}{2}$

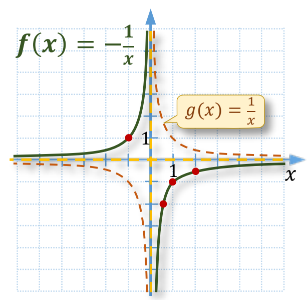


Figure 1.7

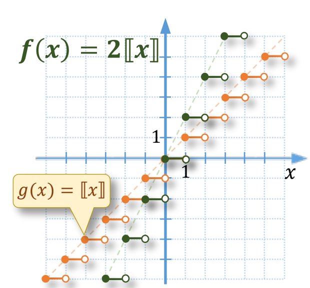
Notice that the graph of function $f(x) = -\frac{1}{x}$ could be obtained by **reflecting** the graph of the basic reciprocal function, $g(x) = \frac{1}{x}$, in the ***x*-axis**.

Function f has the following properties:
Domain: $\mathbb{R} \setminus \{\mathbf{0}\}$
Range: $\mathbb{R} \setminus \{\mathbf{0}\}$
Equations of asymptotes: $\boldsymbol{x = 0}$, $\boldsymbol{y = 0}$
Symmetry: The graph is **symmetrical with respect to the origin**.

b. To graph $f(x) = 2[\![x]\!]$, first, we observe that this is a modified greatest integer function. So, we expect that the graph will consist of half-open segments that line up along a certain line.

Notice that for every x, the value of function f is obtained by multiplying the corresponding value of function $g(x) = [\![x]\!]$ by the factor of 2. Since the segments of the graph of the basic greatest integer function line up along the line $y = x$, we may predict that the segments of the graph of function $f(x) = 2[\![x]\!]$ would line up along the line $y = 2x$. This can be confirmed by calculating and plotting a sufficient number of points, as below.

x	$f(x)$
-0.5	-2
0	0
0.5	0
1	2
1.9	2
2	4



Notice that the graph of function $f(x) = 2[\![x]\!]$ could be obtained by **stretching** the graph of the basic greatest integer function, $g(x) = [\![x]\!]$, in **y-axis** by a factor of **2**.

Function f has the following properties:
Domain: $\mathbb{R}$
Range: $\mathbb{Z}$
The segments of the graph line up along the line $y = 2x$.

c. To graph $f(x) = \frac{1}{2}|x| - 3$, first, we observe that this is a modified absolute value function. So, we expect a "V" shape for its graph.

Notice that for every x, the value of function f is obtained by multiplying the corresponding value of the basic absolute value function $g(x) = |x|$ by a factor of $\frac{1}{2}$, and then subtracting 3. Observe how these operations impact the vertex $(0,0)$ of the basic "V" shape. Since the y-value of the vertex is zero, multiplying it by a factor $\frac{1}{2}$ does not change its position. However, subtracting 3 from the y-value of zero causes the vertex to move to $(0, -3)$.

After plotting the vertex and a few more points, as computed in the table below, we obtain the final graph, as illustrated in *Figure 1.8*.

x	$f(x)$
-2	$\frac{1}{2}$
-1	1
$-\frac{1}{2}$	2
0	*undefined*
$\frac{1}{2}$	-2
1	-1
2	$-\frac{1}{2}$

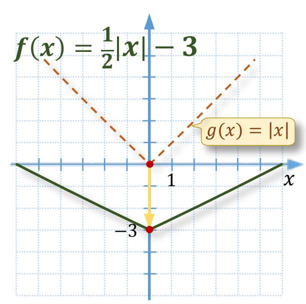


Figure 1.8

Notice that the obtained shape is wider than the shape of the basic absolute value graph. This is because the slopes of the linear sections of the graph are half as steep. So, the function $f(x) = \frac{1}{2}|x| - 3$ could be obtained by

- **compressing** the graph of the basic absolute value function, $g(x) = |x|$, in **y-axis** by a factor of $\frac{1}{2}$, and then
- **translating** the resulting graph by **3 units down**.

Function f has the following properties:
Domain: $\mathbb{R}$
Range: $[\mathbf{-3}, \infty)$
Vertex: $(\mathbf{0}, \mathbf{-3})$
Symmetry: The graph is **symmetrical with respect to the y-axis**.

Generally, to graph a function $\boldsymbol{kf(x)}$, it is enough to **dilate** the graph of $\boldsymbol{f(x)}$, $\boldsymbol{k}$ times in **y-axis**. This dilation is a

- **stretching**, if $|k| > 1$
- **compressing**, if $0 < |k| < 1$
- **flipping** over the **x-axis**, if $k = -1$

Functions of the form $\frac{1}{f(x)}$ or $|f(x)|$

Consider the graphs of a linear function, $f(x) = x + 2$, and its reciprocal, $g(x) = \frac{1}{x+2}$, as illustrated in *Figure 1.9*.

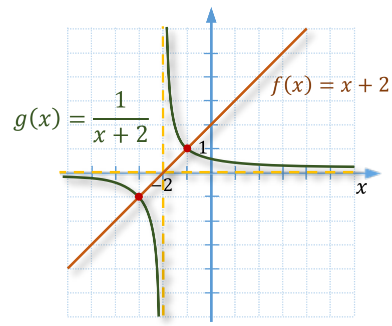


Figure 1.9

Notice that:

- the reciprocal function (in green) has its vertical asymptote at the x-intercept of the linear function (in orange);
- the horizontal asymptote of the reciprocal function $g(x) = \frac{1}{x+2}$ is the x-axis, $y = 0$;
- the points with the y-coordinate equal to 1 or -1 are common for both functions;
- the reciprocal of values close to zero are far away from zero while the reciprocals of values that are far away from zero are close to zero;
- the values of the reciprocal function are of the same sign as the corresponding values of the linear function.

$\boldsymbol{f(x) \to \frac{1}{f(x)}}$

Generally, the graph of the reciprocal of a linear function, $\boldsymbol{g(x) = \frac{1}{ax+b}}$, has the $\boldsymbol{x}$**-axis** as its **horizontal asymptote** and $\boldsymbol{y = -\frac{b}{a}}$ as its **vertical asymptote**.

Now, consider the graphs of the quadratic function $f(x) = x^2 + x - 2$ and the absolute value of this function $g(x) = |x^2 + x - 2|$, as illustrated in *Figure 1.10*.

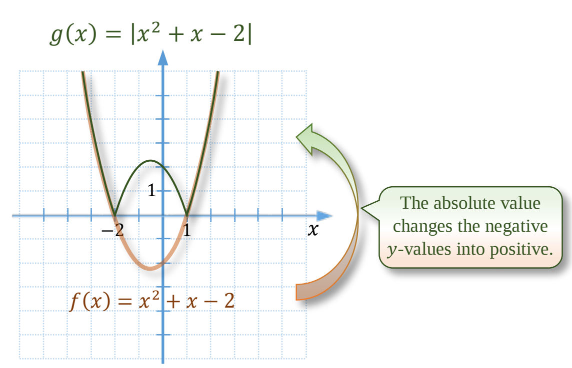


Figure 1.10

Notice that the absolute value function g (in green) follows the original function f (in orange) wherever function f assumes positive values. Otherwise, function g assumes opposite values to function f. So, the graph of the absolute value function g can be obtained by flipping the negative section (section below the x-axis) of the graph of the quadratic function f over the x-axis and leaving the positive sections (above the x-axis) unchanged.

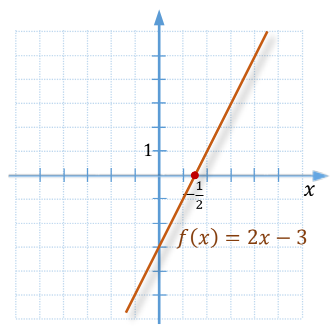
$\boldsymbol{f(x) \to |f(x)|}$

Generally, the graph of the absolute value of any given function $f(x)$, $\boldsymbol{g(x) = |f(x)|}$, can be obtained by **flipping the section(s)** of the graph of the original function f **below the x-axis over the x-axis** and leaving the section(s) above the x-axis unchanged.

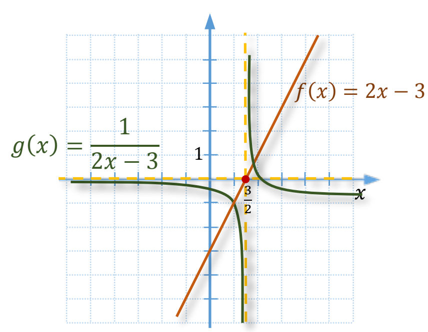
Example 2 ► **Graphing the Reciprocal of a Linear Function**

Using the graph of function $f(x) = 2x - 3$, graph function $g(x) = \frac{1}{f(x)} = \frac{1}{2x+3}$. Determine the x-intercept of function f and the equation of the vertical asymptote of function g.

Solution ► First, we graph function $f(x) = 2x - 3$ as below.



Then, we plot a few 'reciprocal' points. For example, since point $(0, -3)$ belongs to function f, then point $\left(0, -\frac{1}{3}\right)$ must belong to function g. Notice that points $(1, -1)$ and $(2,1)$ are common to both functions, as the reciprocals of -1 and 1 are the same numbers -1 and 1. The graph of function g arises by joining the obtained 'reciprocal' points, as illustrated below.



The equation of the vertical asymptote of the graph of function g is $x = \frac{3}{2}$, and it crosses the x-axis at the x-intercept of function f, which is $\left(\frac{3}{2}, 0\right)$.

Example 3 ► **Graphing the Absolute Value of a Given Function**

Using the graph of function $f(x) = 2x - x^2$, graph function $g(x) = |f(x)| = |2x - x^2|$.

Solution ► To graph function $g(x) = |2x - x^2|$, we may graph function $f(x) = 2x - x^2$ first. Since $2x - x^2 = x(2 - x)$ then the x-intercepts of this parabola are at $x = 0$ and $x = 2$. The first coordinate of the vertex is the average of the two intercepts, so it is 1. Since $f(1) = 1$, then the parabola has its vertex at the point $(1,1)$. So, the graph of function f can be obtained by connecting the two intercepts and the vertex with a parabolic curve. See the orange graph in *Figure 1.11*.

Since $|f(x)| = \begin{cases} f(x) & if \quad f(x) > 0 \\ -f(x) & if \quad f(x) < 0 \end{cases}$, then the graph of function $g(x) = |f(x)|$ is obtained by

- following the (orange) graph of f for the parts where this graph is above the x-axis and
- flipping the parts of the orange graph that lie below the x-axis over the x-axis, as illustrated in green in *Figure 1.11*.

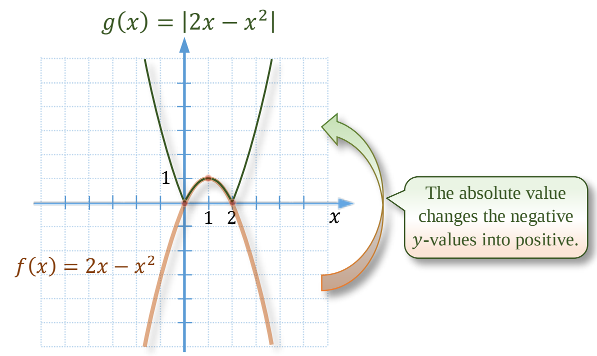


Figure 1.11

Step Function in Applications

The greatest integer function, $[\![x]\!]$, is an example of a larger class of functions, called **step functions**.

Definition 1.2 ► A **step function** is a function whose graph consists of a series of horizontal line segments with jumps in-between them. The line segments can be half-open, open, or closed.

A step function is a constant function on given intervals. However, the value of this function is different for each interval. For example, the function defined as follows:

$f(x) = 1$ for all x-values from the interval $[-2, 1)$,

$f(x) = 2$ for all x-values from the interval $[1, 3)$,

$f(x) = 4$ for all x-values from the interval $[3, 4]$, is a step function with a staircase-like graph, as illustrated in *Figure 1.12*. Such function can be defined with the use of a **piecewise notation**, as below.

$$f(x)=\begin{cases}0, & if \quad -2\le x<1\\ 2, & if \quad 1\le x<3\\ 4, & if \quad 3\le x\le 4\end{cases}$$

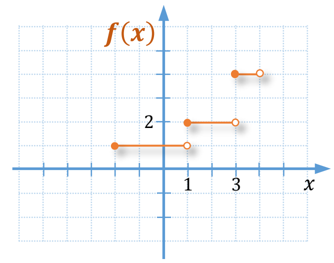


Figure 1.12

Step functions are used in many areas of life, particularly in business. For example, utilities or taxes are often billed according to a step function.

Example 4 ► **Finding a Step Function that Models a Parking Charge**

The cost of parking a car at an airport hourly parking lot is \$5 for the first hour or its portion and \$3 for each additional hour or its portion. Let $C(t)$ represent the cost of parking a car for t hours. Graph $C(t)$ for t in the interval $(0,4]$. Then, using piecewise notation, state the formula for the graphed function.

Solution ► To graph $C(t)$, we may create a table of values first. Observe that

t	$C(t)$
0.5	5
1	5
1.5	$5+3=8$
2	$5+3=8$
2.5	$5+2\cdot 3=11$
3	$5+2\cdot 3=11$
3.5	$5+3\cdot 3=14$
4	$5+3\cdot 3=14$

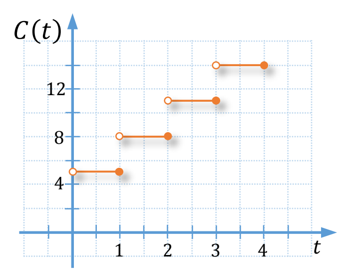
$C(t)=5$ for all t-values from the interval $(0,1]$,

$C(t)=8$ for all t-values from the interval $(1,2]$,

$C(t)=11$ for all t-values from the interval $(2,3]$, and

$C(t)=14$ for all t-values from the interval $(3,4]$.

So, we graph $C(t)$ as below.



Using piecewise notation, function C can be written as

$$C(t)=\begin{cases}5, & if \quad 0<t\le 1\\ 8, & if \quad 1<t\le 2\\ 11, & if \quad 2<t\le 3\\ 14, & if \quad 3<t\le 4\end{cases}$$

C.1 Exercises

Vocabulary Check *Complete each blank with the most appropriate term or phrase from the given list:* ***translating, vertically, dilating, below, above, zeros, step.***

1. The graph of function $f(x-a)$ can be obtained by ______________ the graph of $f(x)$ by a units horizontally.

2. The graph of function $f(x)+b$ can be obtained by translating the graph of $f(x)$ by b units ______________.

3. The graph of function $kf(x)$ can be obtained by _____________ the graph of $f(x)$, k times vertically.

4. The graph of function $|f(x)|$ can be obtained by flipping the section(s) of the graph of $f(x)$ that are _________ the x-axis over the x-axis and leaving unchanged the section(s) that are _________ the x-axis.

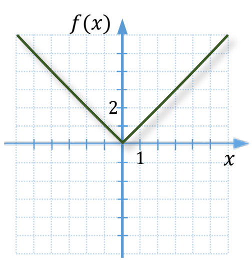
5. The vertical asymptotes of the graph of function $\frac{1}{f(x)}$ are the lines that pass through the ________ of the graph of $f(x)$.

6. The greatest integer function $f(x) = [\![x]\!]$ is an example of a _______ function.

Concept Check

7. Match the name of the basic function provided in **a.-f.** with the corresponding graph in **I-VI**. Then, give the equation of this function and state its domain and range.

a. quadratic **b.** cubic **c.** square root
d. absolute value **e.** greatest integer **f.** reciprocal

I


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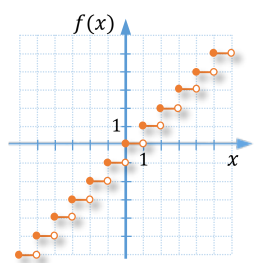

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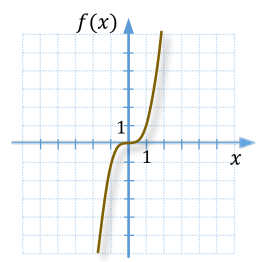

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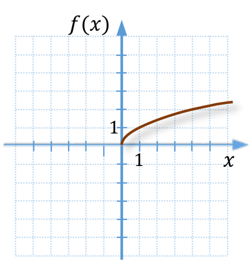

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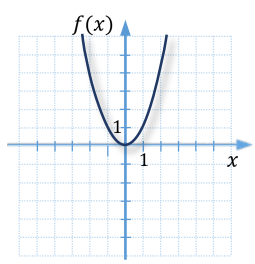
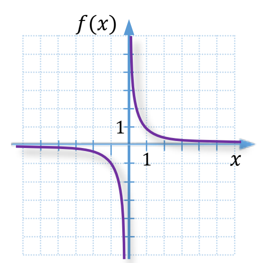


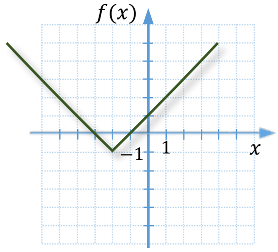
VI


Concept Check

8. Match each absolute value function given in **a.-d.** with its graph in **I-IV**.

a. $f(x) = -|x-1|+2$ **b.** $f(x) = 2|x|-1$ **c.** $f(x) = -|x-2|+1$
d. $f(x) = |x+2|-1$ **e.** $f(x) = -|x|+2$ **f.** $f(x) = 2|x+1|-2$

I



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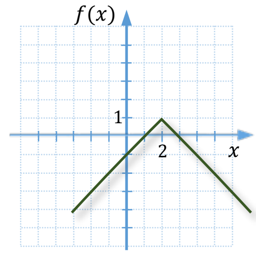

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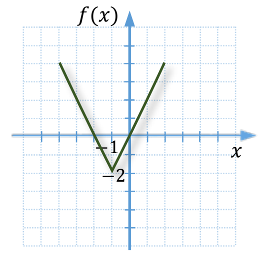

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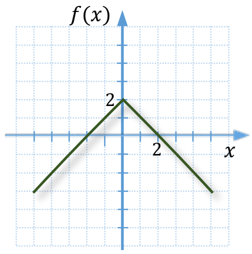

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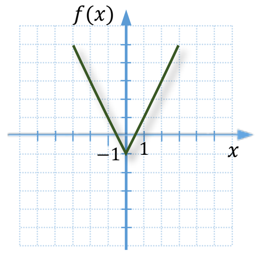
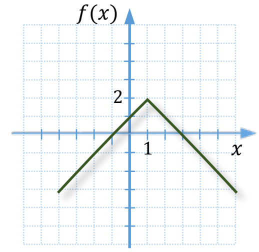


VI



Concept Check

9. How is the graph of $f(x) = \frac{1}{x-5} + 3$ obtained from the graph of $g(x) = \frac{1}{x}$?

10. How is the graph of $f(x) = \sqrt{x+4} - 1$ obtained from the graph of $g(x) = \sqrt{x}$?

Graph each function. Give the domain and range. For rational functions, give the equations of their asymptotes.

11. $f(x) = |x+2|$

12. $f(x) = |x-3|$

13. $f(x) = \sqrt{x} + 2$

14. $f(x) = \sqrt{x} - 3$

15. $f(x) = \frac{1}{x} - 2$

16. $f(x) = \frac{1}{x} + 1$

17. $f(x) = -\frac{2}{x-1}$

18. $f(x) = \frac{1}{x+3} - 2$

19. $f(x) = -\sqrt{x+3}$

20. $f(x) = -(x+2)^3 + 1$

21. $f(x) = 2(x+3)^2 - 4$

22. $f(x) = 2|x+1| - 3$

Concept Check *Evaluate each expression.*

23. $[\![2.1]\!]$

24. $[\![-2.1]\!]$

25. $-[\![2.1]\!]$

26. $-[\![-1.9]\!]$

Graph each function.

27. $f(x) = -[\![x]\!]$

28. $f(x) = [\![x]\!] - 2$

29. $f(x) = [\![x+3]\!]$

For each function $f(x)$, graph its reciprocal $g(x) = \frac{1}{f(x)}$. Determine the x-intercept of function f and the equation of the vertical asymptote of function g.

30. $f(x) = -x$

31. $f(x) = \frac{1}{2}x - 2$

32. $f(x) = -2x + 1$

33. $f(x) = x + 3$

34. $f(x) = -x + 2$

35. $f(x) = 4x - 3$

For each function $f(x)$, graph its absolute value $g(x) = |f(x)|$.

36. $f(x) = x^2 - 4$

37. $f(x) = (2 - x)(x + 3)$

38. $f(x) = 2x^2 + 3x$

39. $f(x) = (2x + 1)(x - 3)$

40. $f(x) = -2x^2 - 5x$

41. $f(x) = x^2 + 3x - 4$

Analytic Skills *Solve each problem.*

42. A certain long-distance carrier provides service between Podunk and Nowhereville. If x represents the number of minutes for the call, where $x > 0$, then the function f defined by

$$f(x) = 0.40[\![x]\!] + 0.75$$

gives the total cost of the call in dollars.

a. Find the cost of a 5.5-minute call

b. Find the cost of a 20.75-minute call.

43. The cost of parking a car at an airport hourly parking lot is $3 for the first half-hour or its portion and $2 for each additional half-hour or its portion. Let $C(t)$ represent the cost of parking a car for t hours. Graph $C(t)$ for t in the interval $(0, 2]$. Then, using piecewise notation, state the formula for the graphed function.

44. An overnight delivery service charges $25 for a package weighing up to 2 kg. For each additional kilogram or its portion, an additional $3 is charged. Let $D(x)$ represent the cost to send a package weighing x kilograms. Graph $D(x)$ for x in the interval $(0, 5]$. Then, using piecewise notation, state the formula for the graphed function.

45. A furniture store pays employees a bonus based on their monthly sales. For sales of $5,000 up to $15,000, the bonus is $500. For sales of $15,000 up to $20,000, the bonus is $800. For sales of $20,000 or more, the bonus is $1,000. Let $B(m)$ represent the amount of bonus received for the monthly sales m. Graph $B(m)$ for m in the interval $(0, 30000]$. Then, using piecewise notation, state the formula for the graphed function.