Scientific Notation

Math 076

Scientific Notation

Writing Large Numbers Using Scientific Notation

Often, scientists need to deal with very large numbers. Rather than repeatedly writing out numbers that have a lot of zeros after them, they will write these numbers using powers of 10 to make things less tedious.

For example, instead of writing the number 32,000,000,000,000,000,000,000 we can move the decimal* over 22 places to the left and multiply by 10^{22}. This looks like:

$$32{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 3.2 \times 10^{22}$$

* Note that although no decimal shows up in this number as it is currently written, it exists after the last zero.

By convention, scientists and mathematicians choose to write the numbers so that there is only one non-zero digit to the left of the decimal place.

Example 1: Write 9,250,000,000,000,000 using scientific notation.

Solution: We need to move the decimal point to the left 15 places:

$$9{,}250{,}000{,}000{,}000{,}000 = 9.25 \times 10^{15}$$

Example 2: Write 63,120,000,000,000,000,000 in scientific notation

Solution: We need to move the decimal point to the left 19 places:

$$63{,}120{,}000{,}000{,}000{,}000{,}000 = 6.312 \times 10^{19}$$

Similarly, we can write very small numbers in scientific notation by moving the decimal place to the right. This is like dividing by 10 so the exponent in scientific notation will be negative.

Example 3: Write 0.00000000054 in scientific notation.

Solution: In order to have one non-zero digit before the decimal place we need to move the decimal to the right 10 places. This gives us:

$$0.00000000054 = 5.4 \times 10^{-10}$$

Example 4: Write 0.0000000123 in scientific notation.

Solution: We need to move the decimal point right 8 places. This gives:

$$0.0000000123 = 1.23 \times 10^{-8}$$

Converting from Scientific Notation to Standard Form

We also need to be able to take numbers in scientific notation and get them back to standard form.

Example 5: Write 5.5×10^6 in standard form.

Solution: Note that because the exponent in this case is positive, this is really a large number. So, to convert back to standard form we know that we will be moving the decimal point 6 places to the right. We will need to add some zeros back into the extra places to accomplish this.

$$5.5 \times 10^6 = 5{,}500{,}000$$

Example 6: Write 9.81×10^{12} in standard form.

Solution: The decimal point needs to move 12 places to the right. Again, add zeros as necessary to the empty places.

$$9.81 \times 10^{12} = 9{,}810{,}000{,}000{,}000$$

If you are in doubt as to whether your answer is correct, you can always check by converting back to scientific notation.

Example 7: Write 5.5×10^{-7} in standard form.

Solution: This time the exponent is negative. So, we know that the original number is very small. This means that the decimal point is going to need to move 7 places to the left in order to get the number in standard form.

$$5.5 \times 10^{-7} = 0.00000055$$

Example 8: Write 1.253×10^{-10} in standard form.

Solution:

$$1.253 \times 10^{-10} = 0.0000000001253$$

Multiplying and Dividing Numbers in Scientific Notation

Recall from our exponent laws that when you multiply powers with the same base, you add the exponents. For example:

$$10^5 \times 10^7 = 10^{12}$$

$$\text{and,} \quad 10^8 \times 10^{-4} = 10^4$$

Also, when you are dividing powers with the same base, you subtract the exponents. For example:

$$\frac{10^7}{10^5} = 10^2$$

$$\frac{10^{10}}{10^{-3}} = 10^{13}$$

The same principle applies when we are multiplying two numbers in scientific notation.

Example 9: Multiply $(1.2 \times 10^{10})(2.6 \times 10^{15})$

Solution: Multiply the numbers to figure out the number component of the answer and add the exponents of the 10's to figure out the exponent in the answer:

$$(1.2 \times 10^{10})(2.6 \times 10^{15}) = 3.12 \times 10^{25}$$

Example 10: Multiply $(2.35 \times 10^{14})(3.5 \times 10^{-9})$

Solution: $(2.35 \times 10^{14})(3.5 \times 10^{-9}) = 8.225 \times 10^{5}$

Sometimes, the answer may come out with more than one digit to the left of the decimal. In scientific notation, your answer should only have one digit to the left of the decimal. If this happens, you simply need to adjust by moving the decimal over one more place and adding one to the exponent.

Example 12: Multiply $(8.5 \times 10^{24})(5 \times 10^{6})$

Solution: $(8.5 \times 10^{24})(5 \times 10^{6}) = 42.5 \times 10^{30}$
$= 4.25 \times 10^{31}$

Note that there should not be 2 digits before the decimal. Move the decimal to the left one place and add one to the exponent.

Dividing numbers in scientific notation is very similar. Simply divide the numbers and subtract the exponents of the 10's.

Example 13: $\frac{8.5\times10^{12}}{2.5\times10^{8}} = \mathbf{3.4 \times 10^{4}}$

Example 14: $\frac{6.5\times10^{5}}{5\times10^{15}} = \mathbf{1.3 \times 10^{-10}}$

Sometimes, the answer won't come out in correct scientific notation and we need to adjust by moving the decimal. If there isn't exactly one non-zero digit before the decimal, you can move the decimal place one spot to the right and subtract one from the exponent of the 10.

Example 15: $\frac{2.5\times10^{22}}{5\times10^{15}} = \mathbf{0.5 \times 10^{7}}$
$= 5 \times 10^{6}$

Note that there needs to be one non-zero digit before the decimal. Move the decimal to the right one place and subtract one from the exponent.

Exercises

1. Write each number in scientific notation:
   a. 6,230,000,000
   b. 510,000,000,000,000
   c. 0.0000056
   d. 0.0000000009

2. Write each number in standard form:
   a. 5×10^5
   b. 9.36×10^{-6}
   c. 8.33×10^{15}
   d. 2.1×10^{-12}

3. Evaluate each expression and write your answer in scientific notation:
   a. $(3 \times 10^5)(1.3 \times 10^4) =$
   b. $(9.5 \times 10^7)(4.5 \times 10^3) =$
   c. $\frac{8\times10^9}{4\times10^7} =$
   d. $(2.54 \times 10^{-5})(1.7 \times 10^{10}) =$
   e. $\frac{(3.6\times10^9)(2\times10^9)}{1.2\times10^7} =$
   f. $\frac{6\times10^9}{5\times10^{-7}} =$

Solutions

1. a) 6.23×10^9 b) 5.1×10^{14} c) 5.6×10^{-6} d) 9×10^{-10}
2. a) 500,000 b) 0.00000936 c) 8,330,000,000,000,000 d) 0.0000000000021
3. a) 3.9×10^9 b) 4.275×10^{11} c) 2×10^2 d) 4.318×10^5

   e) 6×10^{11} f) 1.2×10^{16}