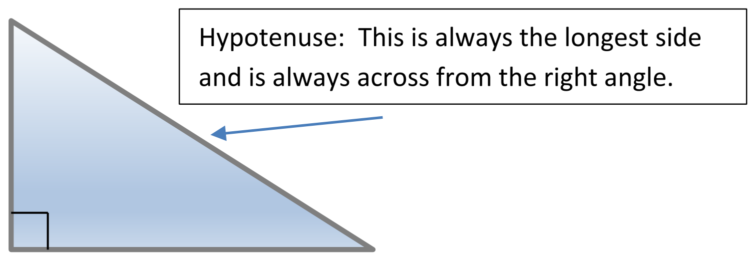
# Trigonometry

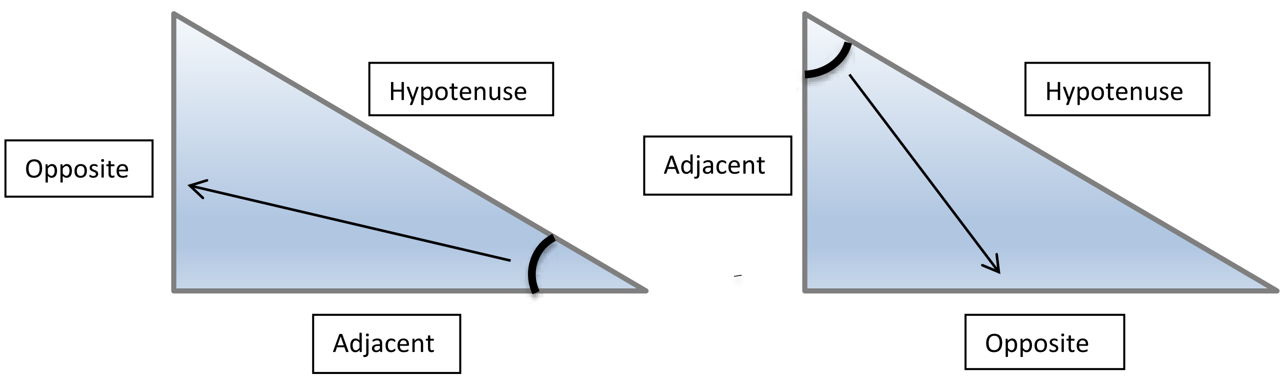
*Math 076*

# Right Angle Trigonometry

Trigonometry provides us with a way to relate the length of sides of a triangle to the measure of its angles. There are three important trigonometric functions that we will learn about in this booklet. They are sine, cosine, and tangent. Each of them represents a ratio between two different sides of a right triangle. To begin, we need to learn how to name the sides of a right triangle:




The other two sides are called "opposite" and "adjacent". Which one is which depends on the angle we are dealing with. "Opposite" is across the triangle from the angle you are dealing with while the "adjacent" side is beside the angle that you are dealing with.




In general, we just use the letters O, A, and H to represent "opposite", "adjacent", and "hypotenuse" respectively rather than writing the words out every time.

There are three formulas that relate the ratios of sides to the size of the angles.

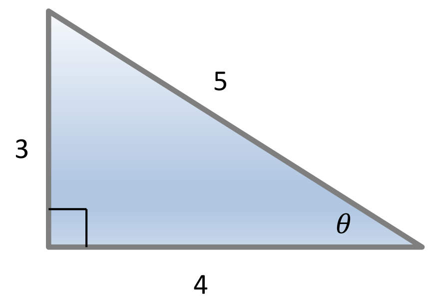
Call the angle that we are dealing with $\theta$ (We generally use Greek letters to represent unknown angles, this symbol is called theta). The formulas that you need to know are:

$$sin\theta = \frac{opposite}{hypotenuse} \qquad cos\theta = \frac{adjacent}{hypotenuse} \qquad tan\theta = \frac{opposite}{adjacent}$$

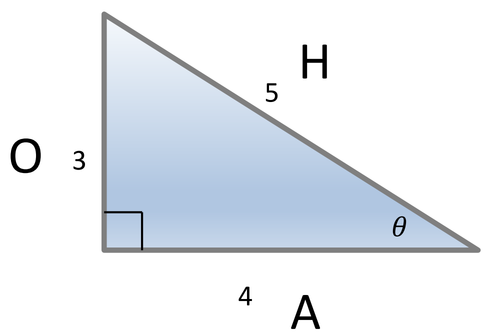
***This can be remembered using the mnemonic SOH CAH TOA ***

**Finding Trig Ratios**

**Example 1:** Write the ratios that represent $sin\theta$, $cos\theta$, and $tan\theta$ for the triangle shown below:

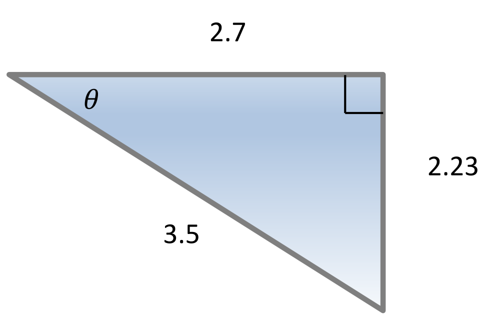



**Solution:** Begin by labeling the sides of the triangle. Then you can setup the ratios using the three formulas. Remember SOH CAH TOA

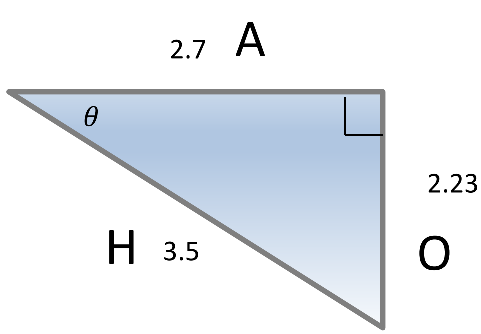



So, $sin\theta = \frac{O}{H} = \frac{3}{5}$ $cos\theta = \frac{A}{H} = \frac{4}{5}$ $tan\theta = \frac{O}{A} = \frac{3}{4}$

**Example 2:** Find $sin\theta$, $cos\theta$, and $tan\theta$ for the triangle shown below. Express your answer to 2 decimal places.




**Solution:** Label the triangle.



$$sin\theta = \frac{O}{H} = \frac{2.23}{3.5} = 0.64 \qquad cos\theta = \frac{A}{H} = \frac{2.7}{3.5} = 0.77 \qquad tan\theta = \frac{O}{A} = \frac{2.23}{2.7} = 0.83$$

**Example 3:** Find $\sin 35°$.

**Solution:** You should know that $\sin 35°$ represents the ratio of $\frac{O}{H}$ in a right triangle where the angle that we are dealing with is $35°$. To calculate it, just use your calculator. It has all three trig functions programmed into it.

$$\sin 35° = 0.574$$

Note: On some calculators you may have to enter the angle FIRST and then push the appropriate trig button. On other calculators, you hit the trig function first, then enter the angle and press enter.

**Example 4:** Calculate each of the following to 3 decimals:

a) $\sin 25° =$
b) $\tan 60° =$
c) $\cos 51° =$

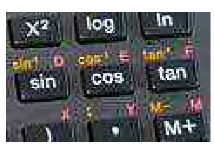
**Solution:** Just punch it into your calculator.

a) $\sin 25° = 0.423$
b) $\tan 60° = 1.732$
c) $\cos 51° = 0.629$

**Finding Angles**

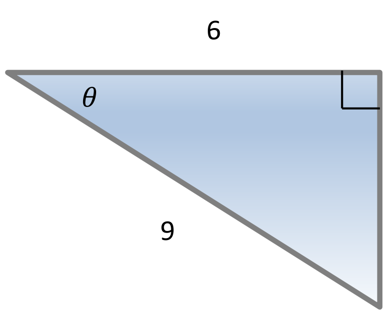
We can use the inverse trig functions on a scientific calculator to calculate unknown angles in a right triangle if we know the lengths of at least 2 sides.

Calculators can be a bit different from each other, but most have buttons that look like this:

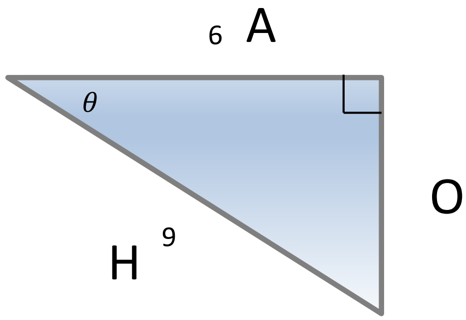



Notice, the $sin^{-1}$, $cos^{-1}$, and $tan^{-1}$ located above the sin, cos, and tan buttons. These are the inverse trig functions. To use them, you will need to use the "2nd" or "INV" button.

**Example 5:** Find the measure of $\theta$ in the following triangle accurate to 1 decimal place:



**Solution:** Begin by labelling the sides so we can figure out which trig function to use.




The two sides that we are dealing with here are A and H. From SOH CAH TOA we know that this means we are dealing with the cosine function.

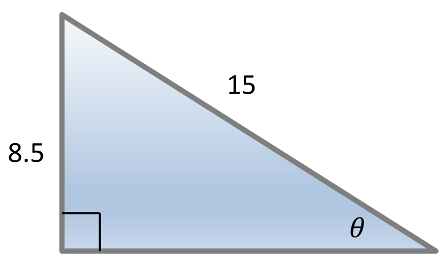
$$\cos\theta = \frac{A}{H}$$

$$\cos\theta = \frac{6}{9}$$

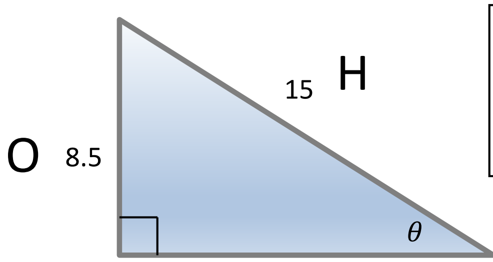
$$\theta = \cos^{-1}\left(\frac{6}{9}\right)$$

Using our calculator we get: $\theta = 48.2°$

**Example 6:** Find the measure of $\theta$ in the following triangle accurate to 1 decimal place:




**Solution:** Always begin by labelling the sides




The two sides that we are dealing with here are O and H. From SOH CAH TOA we know that this means we are dealing with the sine function.

$$\sin\theta = \frac{O}{H}$$

$$\sin\theta = \frac{8.5}{15}$$

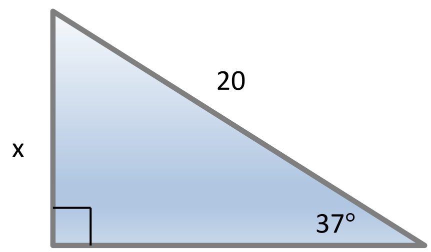
$$\theta = \sin^{-1}\left(\frac{8.5}{15}\right)$$

Using our calculator we get: $\theta = 34.5°$

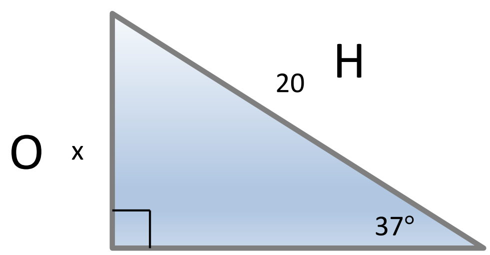
**Finding Sides and Solving Triangles**

Combining these trig functions with a tiny bit of algebra is a powerful tool that allows us, given any two pieces of information about a right triangle, to find the measures of any other parts of the triangle.

**Example 7:** Find the length of side x accurate to two decimal places.




**Solution:** Begin by labelling the sides so we can figure out which trig function to use.




The two sides that we are dealing with are O and H. From SOH CAH TOA we know that we need to use the sine function.

$$\sin\theta = \frac{O}{H}$$

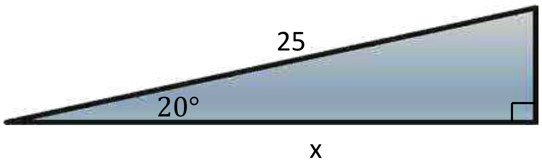
$$\sin 37° = \frac{x}{20}$$

$$20 \times \sin 37° = \frac{x}{20} \times 20$$ Multiply both sides by 20 to solve for x.

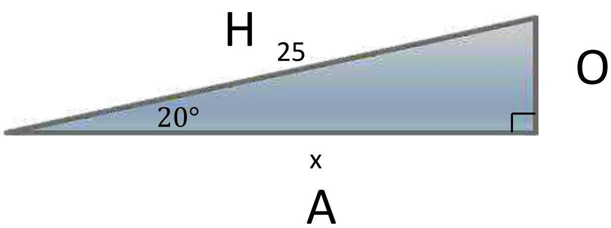
$$20 \times \sin 37° = x$$ Use calculator

$$12.04 = x$$

**Example 8:** Determine the length of side x.




**Solution:** Begin by labeling the sides relative to the angle that we are dealing with (the 20° angle).




The two sides that we know are A and H.

$$cos\theta = \frac{A}{H}$$

We get:

$$\cos(20°) = \frac{x}{25}$$

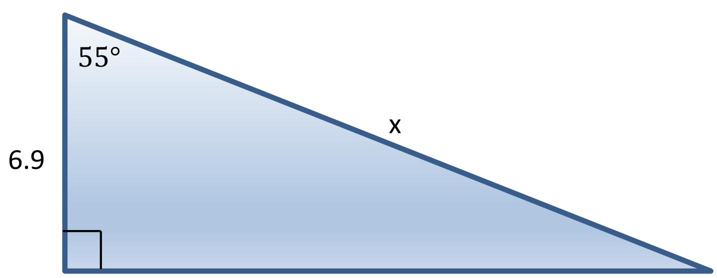
$$25 \times \cos(20°) = \frac{x}{25} \times 25$$

Multiply both sides by 25

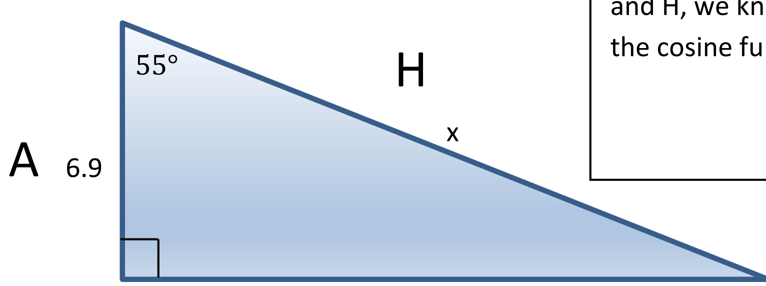
$$25 \times \cos 20° = x$$

$$x = 23.49$$

**Example 9:** Find the length of side x accurate to 2 decimal places.




**Solution:** Again, label the sides.




Because we are dealing with sides A and H, we know that we should use the cosine function.

$$\cos\theta = \frac{A}{H}$$

$\cos 55° = \frac{6.9}{x}$

In order to solve for x, we do not want it on the bottom, multiply both sides by x to get it on the top.

$x \times \cos 55° = \frac{6.9}{\cancel{x}} \times \cancel{x}$

x's cancel. Now divide both sides $\cos 55°$

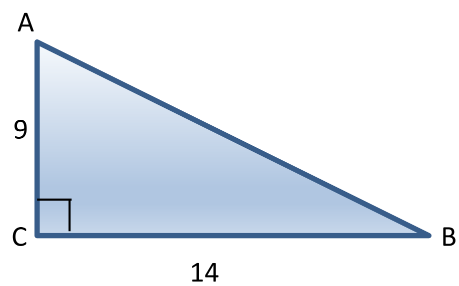
$\frac{x \times \cancel{\cos 55°}}{\cancel{\cos 55°}} = \frac{6.9}{\cos 55°}$

$\cos 55°$ cancels

$$x = \frac{6.9}{\cos 55°}$$

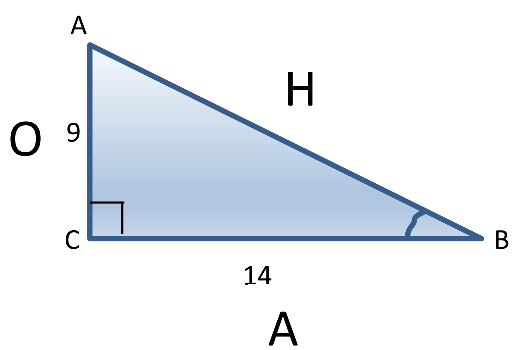
$x = 12.03$

**Example 10:** Solve the following triangle:




Note: "Solving a triangle" means finding the measure of all of the sides and all of the angles. There are multiple ways to approach this. In this example, we will start by finding the measure of angle B.

**Solution:** Always begin by labelling the triangle. Since we are starting by looking for the measure of angle B. Label the triangle from this angle.




Labelling from Angle B we realize that the two sides that we know are the opposite and adjacent. This means that we can find angle B by using the tangent function.

$$tanB = \frac{O}{A}$$

$$tanB = \frac{9}{14}$$

$$B = \tan^{-1}\left(\frac{9}{14}\right)$$

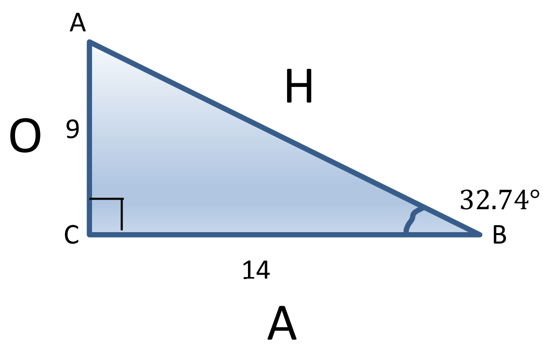
$$B = 32.74°$$

Now, the angles in a triangle add up to 180°. So, we can find the measure of angle A by Subtracting B and C from 180°.

$$A = 180° - 90° - 32.74°$$

$$A = 57.26°$$

Now the only missing piece of information is the length of side AB (the hypotenuse). There are several ways to find this. Now that we know the measure of angle A and B we could use either one to calculate the length of AB. Since we already labelled our triangle from B, let's use that.




Since we need H and we know both O and A we can use either cosine or sine to calculate H.

$$sinB = \frac{O}{H}$$

$$\sin(32.74°) = \frac{9}{H}$$

$$H \times \sin(32.74°) = \frac{9}{\cancel{H}} \times \cancel{H}$$

Multiply both sides by H, the H's on the right cancel. Then we want to get rid of $\sin(32.74°)$ so we divide both sides by it.

$$\frac{H \times \cancel{\sin(32.74°)}}{\cancel{\sin(32.74°)}} = \frac{9}{\sin(32.74°)}$$

$$H = 16.64$$

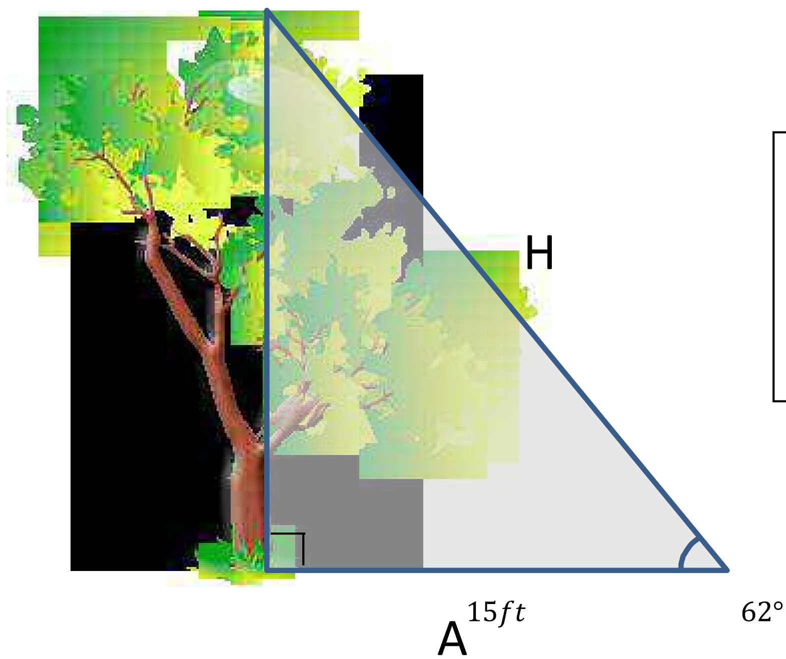
We now know all of the angles and all of the sides of the triangle. We have solved the triangle!

**Trigonometry Word Problems**

To solve word problems involving triangles, we just need to draw a picture that illustrates the problem and then use our trig skills to figure out the missing piece of information. Always start by drawing a picture.

**Example 1:** A tree's shadow makes an angle with the ground that measures 62°. Sarah measures the length of the shadow and finds that it is 15ft long. How tall is the tree?

**Solution:** Start by drawing and labelling a diagram of the situation and label the triangle from the angle that we know. Since we are looking for the height of the tree, let's call that x.




We know A and we are looking for O so SOH CAH TOA tells us that we should use the tangent function

$$tan\theta = \frac{O}{A}$$

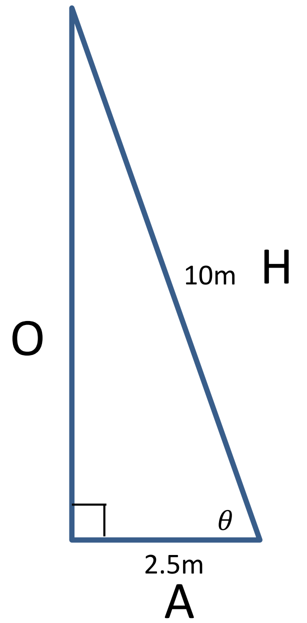
$$\tan 62° = \frac{x}{15}$$

$$15 \times \tan 62° = x$$

$$x = 28.21\ ft$$

**Example 2:** A 10m ladder is leaning against the side of a building. The base of the ladder is 2.5m away from the bottom of the wall. What angle does the bottom of the ladder make with the ground?

**Solution:** Begin by drawing a diagram and labelling it with the relevant information. We will call the angle that we are looking for $\theta$.




We are dealing with sides A and H so we know that we are using the cosine function.

$$cos\theta = \frac{A}{H}$$

$$\cos\theta = \frac{2.5}{10}$$

$$\theta = \cos^{-1}\left(\frac{2.5}{10}\right)$$

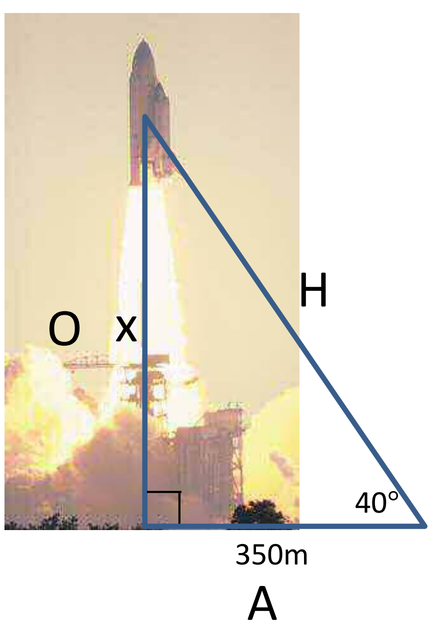
$$\theta = 75.52°$$

**Example 3:** NASA launches a rocket. Their observation station is 350 m away from the launch site. 2 seconds after launch the angle of elevation between their observation station and the rocket is 40°. How high is the rocket at this time?

Note: Angle of Elevation is the measure of an angle going UP from the horizontal.

In contrast, the Angle of Depression is the measure of an angle going below the horizontal.

**Solution:** Again, draw and label a diagram. Call the height of the rocket x.




The two sides that we are dealing with are O and A so we know that we are using the tangent function.

$$\tan\theta = \frac{O}{A}$$

$$\tan 40° = \frac{x}{350}$$

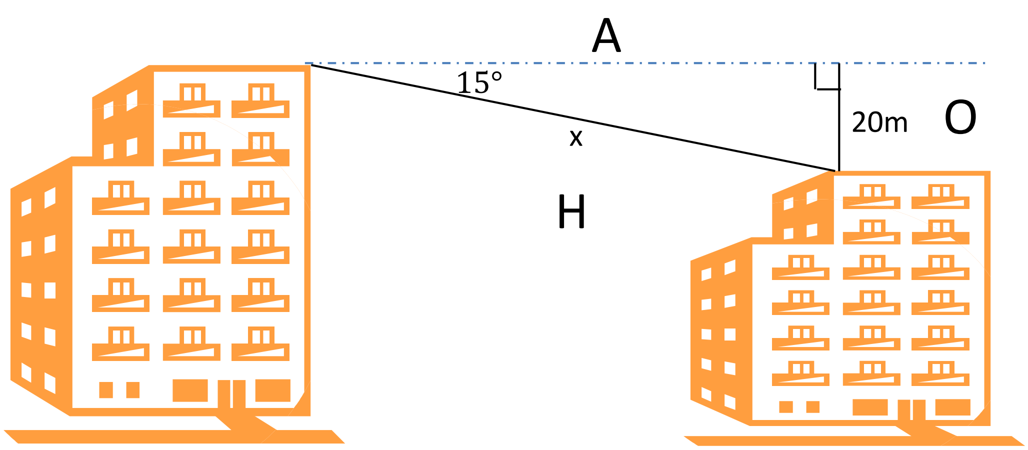
$$350 \times \tan 40° = \frac{x}{\cancel{350}} \times \cancel{350}$$

$$x = 350 \times \tan 40°$$

$$x = 293.68\, m$$

**Example 4:** Natalia needs to hook up a zip line from the top of one building to the top of another shorter building. Standing at the top of the taller building, she measures that the angle of depression between the buildings is 15° and she knows that the shorter building is 40 ft tall and her building is 60 ft tall. How long does her zip line need to be?

**Solution:** Again, draw a diagram to illustrate the situation. Remember that the angle of depression is measured going DOWN from the horizontal. Use this to build our triangle. The difference in the heights of the buildings is 20m. Call the length of the zip line x.




$$\sin 15° = \frac{20}{x}$$

$$x \times \sin 15° = \frac{20}{\cancel{x}} \times \cancel{x}$$

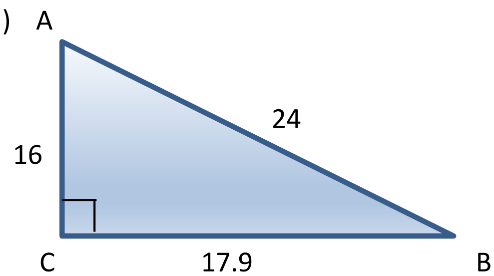
$$\frac{x \times \cancel{\sin 15°}}{\cancel{\sin 15°}} = \frac{20}{\sin 15°}$$

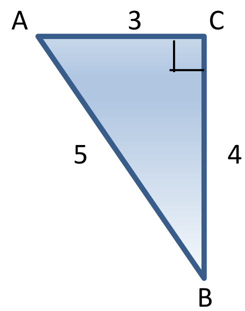
$$x = 77.27 ft$$

**Practice Problems**

1. Identify the largest and smallest angles in each triangle. Also find $\sin B$, $\cos B$, $and$ $\tan B$ for each triangle.

> Note: The smallest angle is always opposite the smallest side. The largest angle is opposite the longest side.

a)



b)


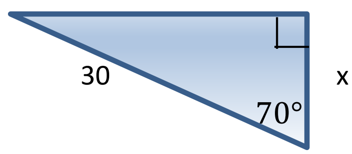

2. Evaluate:

a) $\sin 15° =$

b) $\cos 28° =$

c) $\tan 49° =$

3. Find the unknown side to the nearest tenth.

a)
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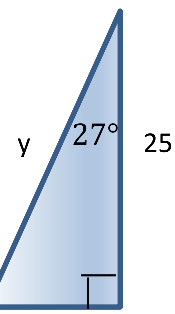
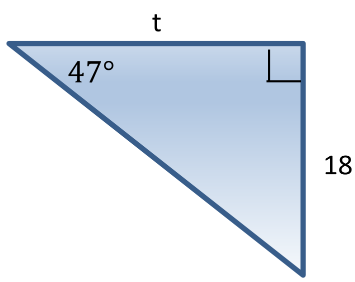



c)
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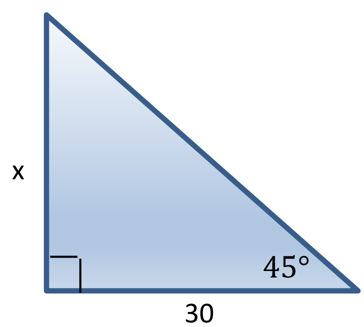


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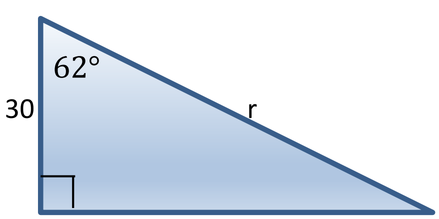


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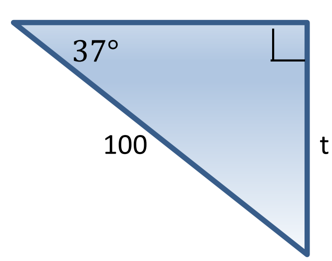
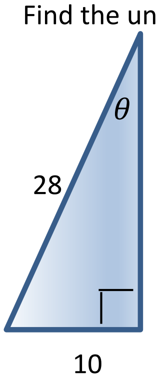
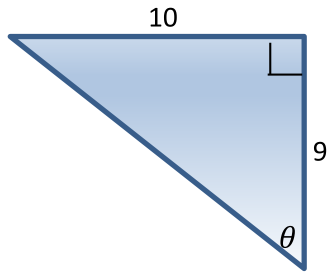


4. Find the unknown angles to the nearest degree.

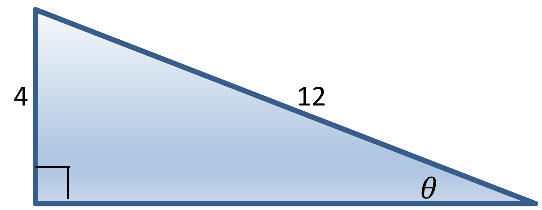
a)



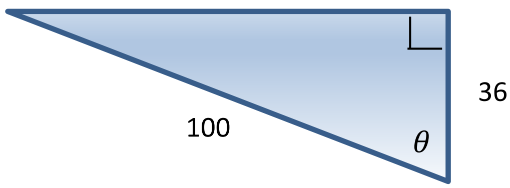

b)



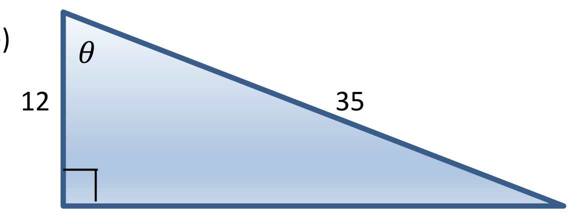

c)




d)




e)

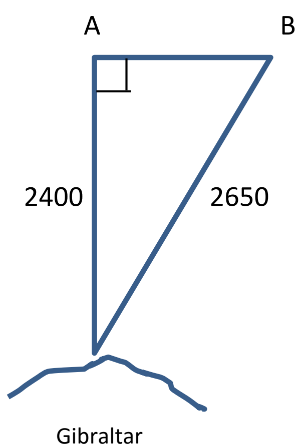



5. Find the length of a tree shadow which has a 38° elevation if the tree is 21 meters tall. (Note: This means that the angle that the shadow makes with the ground is 38°.)

6. The angle of depression from the top of a 15 m building to a car on the street is 40°. How far is the car from the building? (remember: angle of depression is measured *down* from the horizontal)

7. A ship is passing through the Strait of Gibraltar. At its closets point of approach, Gibraltar radar determines that it is 2400 m away. Later, the radar determines that it is 2650m away.

   a) By what bearing $\theta$ did the ship's bearing from Gibraltar change?
   b) How far did the ship travel between the two observations?



8. You lean a ladder 6.7m long against a wall. The ladder makes an angle of 63° with the level ground. How high up is the top of the ladder?

9. You must order new rope for a flagpole. To find out what length of rope is needed, you observe that the pole casts a shadow 11.6m long on the ground. The angle of elevation of the sun is 36.83°. How tall is the pole?

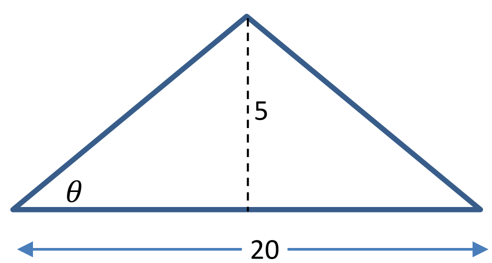
10. Your cat is trapped on a tree branch 6.5m above the ground. Your ladder is only 6.7m long. If you place the tip of your ladder on the tree branch, what angle will it make with the ground?

11. Scientists estimate the heights of features on the moon by measuring the lengths of the shadows they cast on the moon's surface. From a photograph, you find that the shadow cast on the inside of a crater by its rim is 325m long. At the time the photograph was taken, the sun's angle of elevation from this place on the moon's surface was 23.617°. How high does the rim rise above the inside of the crater?

12. Commercial airliners fly at an altitude of about 10km. They start descending toward the airport when they are still far away, so that they will not have to dive at a steep angle.

    a) If the pilot wants the plane's path to make an angle of 3° with the ground, how far from the airport must he start descending?
    b) If he starts descending 300km away from the airport, what angle will the plane's path make with the horizontal?

13. A symmetrical triangular roof has a rise of 5ft and a span of 20 ft. Find the slope angle $\theta$ and the length of the rafters.




14. A greenhouse "shed" roof has rafters which are 4m long. The raters join a 2m high wall to a 5m high wall. Find the angle $\theta$ that the rafter makes with the 5m wall. Also find the width of the building.

15. The angle of elevation from a boat to the top of a lighthouse is 35.7°. If the boat is 48m from the base of the lighthouse, find the height of the lighthouse.

**Solutions**

1a) C – largest, B – smallest, $\sin B = 0.667$ , $\cos B = 0.746, \tan B = 0.894$

b) C – largest, B – smallest, $\sin B = 0.6$ , $\cos B = 0.8, \tan B = 0.75$

2a) 0.259 b) 0.883 c) 1.15 3a) 10.3 b) 28.1 c) 16.8 d) 30 e) 63.9 f) 60.2

4a) $20.9°$ b) $48°$ c) $19°$ d) $69°$ e) $70°$ 5) 26.9m 6) 17.9m 7)a) 25.08° b) 1123.3m

8) 5.97m 9) 8.69m 10) $75.97°$ 11) 142.1m 12a) 190.81 km 12b) $1.91°$

13) $\theta = 26.57°$, 11.18ft 14) $41.41°$, 2.65m 15) 34.5m