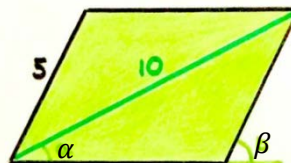**Problems for the 7th Annual Math Match 2020**

1. Show that among any 37 integers that are not divisible by 7, there are 7 integers with the sum divisible by 7.

2. Show that 24 divides $p^2 - 1$ for any prime $p > 3$.

3. Refer to the diagram. Angles α and β are complementary. What is the area of the parallelogram?

4. A circle is inscribed in an isosceles trapezoid with bases a and b. Find the area of this circle.

5. Show that $\sqrt{3 - 2\sqrt{2}} + \sqrt{6 - \sqrt{32}}$ is rational.

6. There are 9 checkers positioned in one of the 3x3 corners of an 8x8 chessboard. You can move a checker by applying central symmetry with respect to any other checker as long as the destination square is empty. Using a finite number of such moves, can you transfer all the checkers to a different 3x3 corner of the chessboard?

7. What is the area of the region of the right triangle, shaded in yellow in the diagram below?

8. Is it possible to divide the lines in the plane into pairs of perpendicular lines so that every line belongs to exactly one pair?