Problems for the 8th Annual Math Match 2023

1. Create a 4-digit number with different digits taken from the set $\{1, 2, \ldots, 9\}$. Then, find the sum of all such 4-digit numbers.

2. Consider a set of n different lines in a plane such that each pair of lines intersects but no three lines intersect at the same point. Show that the lines cut the plane into $\frac{1}{2}(n^2 + n + 2)$ parts.

3. Show how to find the sum
$$2 + 22 + 222 + \cdots + \underbrace{222\ldots2}_{2022\ times}$$

4. Show that any convex polygon can be enclosed by a rectangle with area not larger than twice the area of this polygon.

5. Find the area of a cyclic octagon with four consecutive sides of length 1 and the remaining four sides of length 2.

6. What factor of the form $i!$ should we remove from $1!2!3!\ldots99!100!$ to make a perfect square number?

7. Find all integers n, $1 \leq n \leq 300$ for which n^n is a perfect cube number. How many of such numbers are there?

8. Find all polynomials $P(x)$ satisfying the condition $xP(x-1) = (x-2)P(x)$ for all real x.