Quadratic Equations and Functions - ANSWERS

Q.1 Exercises

1. False

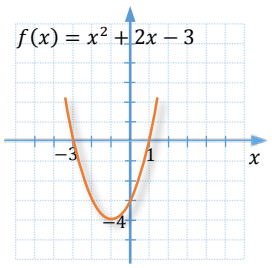
3. True

5. False

7. False

9. **a)**

x	$f(x)$
1	0
0	-3
-1	-4
-2	-3
-3	0



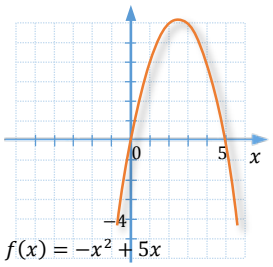
b) $(-3,0)$, $(1,0)$

c) $x \in \{-3,1\}$

The solutions are the first coordinates of the x-interceptes.

11. **a)**

x	$f(x)$
0	0
1	4
2	6
$\frac{5}{2}$	6.25
3	6
4	4
5	0



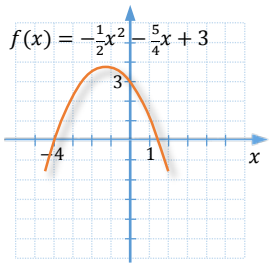
b) $(0,0)$, $(5,0)$

c) $x \in \{0,5\}$

The solutions are the first coordinates of the x-interceptes.

13. **a)**

x	$f(x)$
-4	0
-2	3.5
-1	3.75
0	3
1	1.25
2	-1.5



b) $(-4,0)$, $\left(\frac{3}{2},0\right)$

c) $x \in \left\{-4, \frac{3}{2}\right\}$

The solutions are the first coordinates of the x-interceptes.

15. $x \in \{-4\sqrt{2}, 4\sqrt{2}\}$

17. $n \in \{-2\sqrt{6}, 2\sqrt{6}\}$

19. $y \in \{-2\sqrt{10}, 2\sqrt{10}\}$

21. $x \in \{-7,1\}$

23. $t \in \left\{\frac{-2-2\sqrt{3}}{5}, \frac{-2+2\sqrt{3}}{5}\right\}$

25. $y \in \{-4 - 2\sqrt{11}, -4 + 2\sqrt{11}\}$

27. $y \in \left\{\frac{44}{5}, \frac{56}{5}\right\}$

29. No solution

31. $x \in \left\{\frac{1-3\sqrt{2}}{2}, \frac{1+3\sqrt{2}}{2}\right\}$

33. $y \in \{0,3\}$

35. $n = -2$

37. $y \in \left\{\frac{-7-\sqrt{53}}{2}, \frac{-7+\sqrt{53}}{2}\right\}$

39. No solution

41. $x \in \{6-2\sqrt{5}, 6+2\sqrt{5}\}$

43. $x \in \left\{\frac{-1-\sqrt{7}}{3}, \frac{-1+\sqrt{7}}{3}\right\}$

45. $x \in \left\{\frac{4-\sqrt{3}}{3}, \frac{4+\sqrt{3}}{3}\right\}$

47. $x \in \left\{\frac{2-\sqrt{3}}{3}, \frac{2+\sqrt{3}}{3}\right\}$

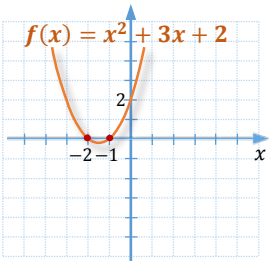
49. $x \in \left\{\frac{1-2\sqrt{19}}{5}, \frac{1+2\sqrt{19}}{5}\right\}$

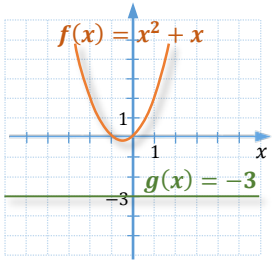
51. $x \in \{1-2\sqrt{2}, 1+2\sqrt{2}\}$

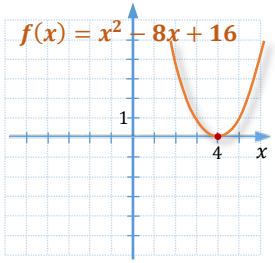
53. $x \in \{-2, -1\}$

55. No solution

57. $x = 4$







59. $a = 1-\sqrt{5} \approx -1.24$, or $a = 1+\sqrt{5} \approx 3.24$

61. $x = \frac{-5-\sqrt{11}}{2} \approx -4.16$, or $x = \frac{-5+\sqrt{11}}{2} \approx -0.84$

63. $y = \frac{-1-\sqrt{7}}{6} \approx -0.27$, or $y = \frac{-1+\sqrt{7}}{6} \approx 0.61$

65. $x = \frac{17-\sqrt{249}}{10} \approx 0.12$, or $x = \frac{17-\sqrt{249}}{10} \approx 3.28$

67. $x = \frac{5-\sqrt{7}}{6} \approx 0.39$, or $x = \frac{5+\sqrt{7}}{6} \approx 1.27$

69. 2 rational solutions; factoring possible

71. 2 real solutions; use quadratic formula

73. 1 double rational solution; factoring possible

75. $k = 25$

77. No, as the product of a rational and irrational number is irrational. This contradicts the fact that the quadratic equation has only integers for its coefficients.

79. $x = 1 \pm \sqrt{10}$

81. $x = \frac{5 \pm 2\sqrt{6}}{2}$

83. $x \in \{-3, 2\}$

85. No solution

87. $x \in \left\{-\frac{3}{2}, 1\right\}$

89. $x = 5 \pm \sqrt{53}$

Q.2 Exercises

1. quadratic in form

3. Quadratic Formula

5. exact; approximate

7. $x \in \{-\sqrt{3}, \sqrt{3}\}$

9. $x \in \{1, 81\}$

11. $y \in \left\{-\frac{1}{2}, \frac{1}{9}\right\}$

13. $t = 1$

15. $x \in \left\{\frac{-5-\sqrt{41}}{2}, -3, -2, \frac{-5+\sqrt{41}}{2}\right\}$

17. $x = 125$

19. $p \in \left\{\frac{-1-\sqrt{5}}{4}, \frac{-1+\sqrt{5}}{4}\right\}$

21. $x \in \{1, 6\}$

23. $v = \pm\sqrt{\frac{Fr}{m}}$

25. $r = \pm\sqrt{\frac{A}{4\pi}}$

27. $r = \pm\sqrt{\frac{Gm_1m_2}{F}}$

29. $b = \pm\sqrt{c^2 - a^2}$

31. $r = \frac{-\pi s \pm \sqrt{\pi^2 s^2 + 4\pi A}}{2\pi}$

33. $t = \frac{-v_0 \pm \sqrt{{v_0}^2 + 2gs}}{g}$

35. $r = -1 \pm \frac{\sqrt{AP}}{P}$ or $r = \frac{-P \pm \sqrt{AP}}{P}$

37. $s = -\frac{t}{3}$ or $s = \frac{t}{2}$

39. $c = \frac{vm}{\sqrt{m^2 - m_0^2}}$ or $c = \frac{vm\sqrt{m^2 - m_0^2}}{m^2 - m_0^2}$

41. $l = \frac{1+\sqrt{5}}{2}w$

43. **a.** $\frac{1}{n}$ **b.** $\frac{h}{n}$

45. 80 mi and 150 mi

47. 24 m and 7 m

49. 30 mi and 40 mi

51. 57 in by 30 in

53. $AC = 5$ or $AC = 14$

55. 4 ft

57. $\frac{33-\sqrt{369}}{4} \approx 3.4$ cm

59. 21 in by 23 in

61. 350 mph; 400 mph

63. 60 mph and 50 mph

65. ~2.56 mph

67. ~254 mph

69. ~3.6 hr

71. Ron: 3.1 hr; Chad: 4.3 hr

73. 9 hr

75. 260

Q.3 Exercises

1. vertex

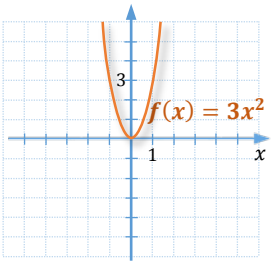
3. shape

5. horizontal

7. minimum; maximum

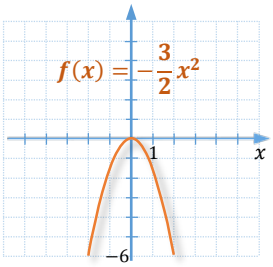
9. **a.-III**; **b.-IV**; **c.-I**; **d.-II**

11. narrower; opens up



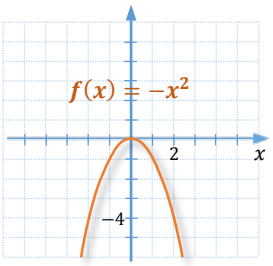
Range $= [0, \infty)$

13. narrower; opens down



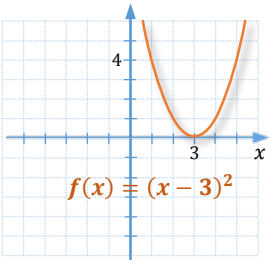
Range $= (-\infty, 0]$

15. same; opens down



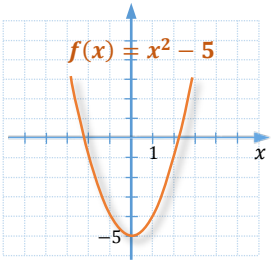
Range $= (-\infty, 0]$

17. shift 3 units to the right



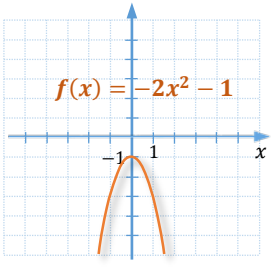
Domain = $\mathbb{R}$
Range $= [0, \infty)$
Axis of symmetry: $x = 3$

19. shift 5 units down



Domain = $\mathbb{R}$
Range $= [-5, \infty)$
Axis of symmetry: $x = 0$

21. S_x; vertical dilation by 2; shift 2 units down

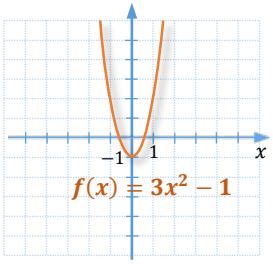


Domain = $\mathbb{R}$
Range $= (-\infty, -1]$
Axis of symmetry: $x = 0$

23. vertex = $(0, -1)$

shape of $3x^2$; opens up

axis of sym.: $x = 0$



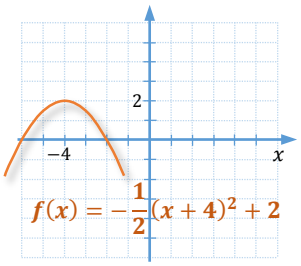
vertical dilation by 3;

shift 1 unit down

25. vertex = $(-4, 2)$

shape of $\frac{1}{2}x^2$; opens down

axis of sym.: $x = -4$



S_x; vertical dilation by $\frac{1}{2}$;

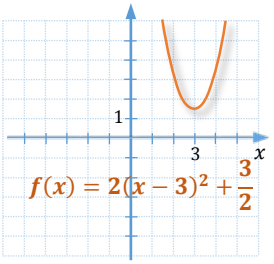
shift 4 units to the left;

shift 2 units up

27. vertex = $(3, \frac{3}{2})$

shape of $2x^2$; opens up

axis of sym.: $x = 3$



vertical dilation by 2;

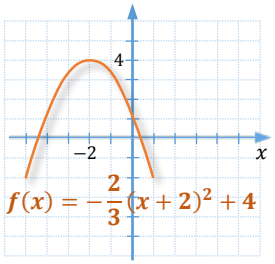
shift 3 units to the right;

shift $\frac{3}{2}$ units up

29. vertex = $(-2, 4)$

shape of x^2; opens down

axis of sym.: $x = -2$



S_x; vertical dilation by $\frac{2}{3}$;

shift 2 units to the left;

shift 4 units up

31. Correct graph: *Student* ***B***

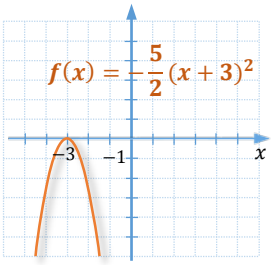
Student ***A****:* incorrect order (dilation should be before the shift)

Student ***C****:* incorrect order (dilation should be before the shift)

Student ***D****:* incorrect order (flip should be before the vertical shift)

Yes, there are other sequences of transformations.

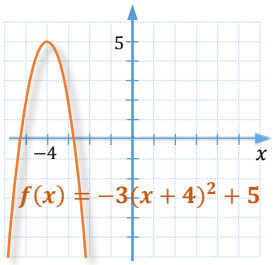
33. vertex = $(-3, 0)$



Maximum value $= 0$;

Range $= (-\infty, 0]$

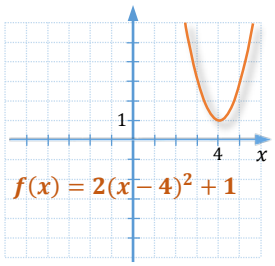
35. vertex = $(-4, 5)$



Maximum value $= 5$;

Range $= (-\infty, 5]$

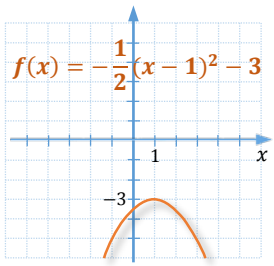
37. vertex = $(4, 1)$



Minimum value $= 1$;

Range $= [1, \infty)$

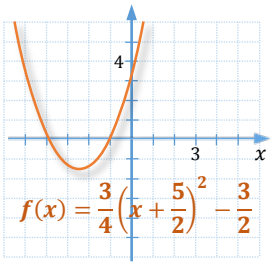
39. vertex = $(1, -3)$



Maximum value $= -3$;

Range $= (-\infty, -3]$

41. vertex = $\left(-\frac{5}{2}, -\frac{3}{2}\right)$



Minimum value $= -\frac{3}{2}$;
Range $= \left[-\frac{3}{2}, \infty\right)$

43. $f(x) = -(x-3)^2 + 5$

45. $f(x) = \frac{1}{2}x^2 + 2$

47. $f(x) = -\frac{1}{3}(x-1)^2 + 2$

Q.4 Exercises

1. vertex

3. minimum

5. second; first

7. average

9. $f(x) = (x-2)^2 - 9;\ V(2,-9)$

11. $f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{27}{4};\ V\left(\frac{1}{2}, \frac{27}{4}\right)$

13. $f(x) = 2(x-1)^2 - 1;\ V(1,-1)$

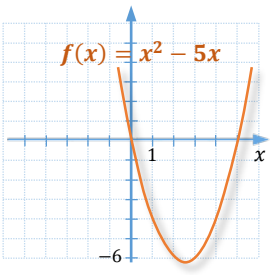
15. $f(x) = -2(x+2)^2 + 18;\ V(-2,18)$

17. $V(-3,-6)$

19. $V(4,-15)$

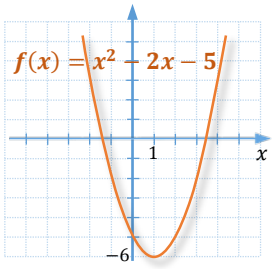
21. $V\left(\frac{7}{10}, -\frac{49}{20}\right)$

23. $V\left(\frac{5}{2}, -\frac{25}{4}\right)$; opens up
shape of x^2



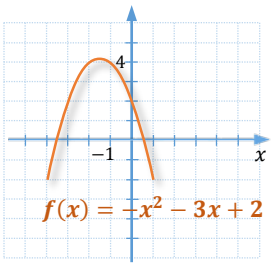
$D = \mathbb{R}$; Range $= \left[-\frac{25}{4}, \infty\right)$

25. $V(1,-6)$; opens up
shape of x^2



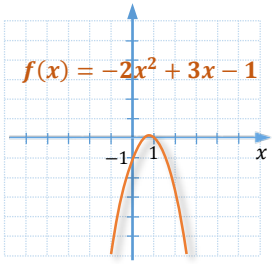
$D = \mathbb{R}$; Range $= [-6, \infty)$

27. $V\left(-\frac{3}{2}, \frac{17}{4}\right)$; opens down
shape of x^2



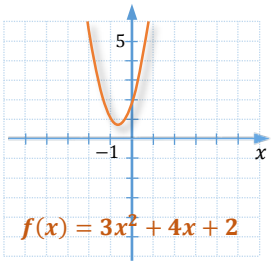
$D = \mathbb{R}$; Range $= \left(-\infty, \frac{17}{4}\right]$

29. $V\left(\frac{3}{4},\frac{1}{8}\right)$; opens down
shape of $2x^2$



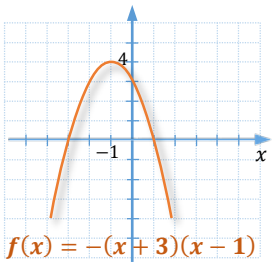
$D = \mathbb{R}$; Range $= \left(-\infty, \frac{1}{8}\right]$

31. $V\left(-\frac{2}{3},\frac{2}{3}\right)$; opens up
shape of $3x^2$



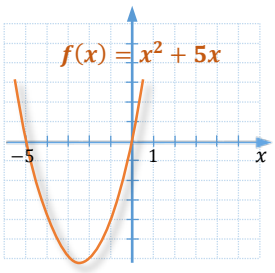
$D = \mathbb{R}$; Range $= \left[\frac{2}{3}, \infty\right)$

33. zeros: -3, 1; $V(-1,4)$;
opens down; shape of x^2



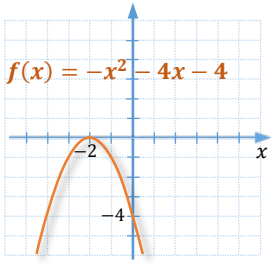
Maximum value $= 4$
Maximum occurs at $x = -1$

35. zeros: -5, 0; $V\left(-\frac{5}{2},-\frac{25}{4}\right)$;
opens up; shape of x^2



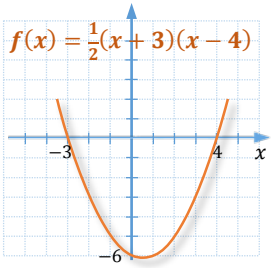
Minimum value $= -\frac{25}{4}$
Minimum occurs at $x = -\frac{5}{2}$

37. zero: -2; $V(-2,0)$;
opens down; shape of x^2



Maximum value $= 0$
Maximum occurs at $x = -2$

39. zeros: -3, 4; $V\left(\frac{1}{2},-\frac{49}{8}\right)$;
opens up; shape of $\frac{1}{2}x^2$



Minimum value $= -\frac{49}{8}$
Minimum occurs at $x = \frac{1}{2}$

41. $x = \frac{x_1 + x_2}{2}$ **43.** false **45.** true **47.** false

49. false **51.** 2500 ft; 25 sec **53.** 20; \$210 **55.** 20, -20

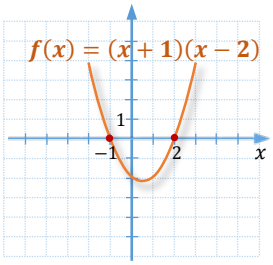
57. 24 ft by 32 ft **59.** **a.** $p(n) = 50 - n$ **b.** $R(n) = (50 - n)n$ **c.** 25; \$625

Q.5 Exercises

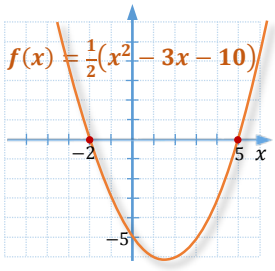
1. false **3.** false **5.** true

7. **a.** $[-1,3]$ **b.** $(-\infty,-1) \cup (3,\infty)$ **9.** **a.** $(-\infty,-1] \cup \{3\}$ **b.** $(-1,3) \cup (3,\infty)$

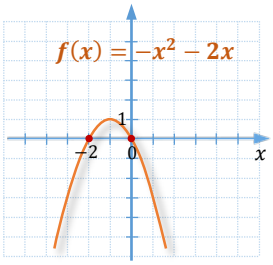
11. $x \in (-1,2)$



13. $x \in [-2,5]$



15. $x \in (-\infty,-2) \cup (0,\infty)$



17. $x \in (-4,5)$

19. $x \in (-\infty,-3] \cup [4,\infty)$

21. $x \in \left(-2,-\frac{1}{2}\right)$

23. $x = 2$

25. $x \in (-\infty,-5) \cup (4,\infty)$

27. $x \in (-\infty,-3] \cup [2,5]$

29. $x \in \left[0,\frac{1}{2}\right] \cup [2,\infty)$

31. $x \in (-\infty,-1) \cup (0,\infty)$

33. $x \in \left(-3,\frac{1}{2}\right]$

35. $y \in (-5,-2)$

37. $x \in \left(-\infty,-\frac{7}{2}\right] \cup (-2,\infty)$

39. $t \in \left(-\infty,-\frac{15}{2}\right] \cup (-3,\infty)$

41. $y \in \left(-\infty,-\frac{5}{2}\right) \cup \left[-\frac{3}{2},\infty\right)$

43. $x \in \left[0,\frac{1}{2}\right) \cup \left[\frac{5}{2},\infty\right)$

45. $x \in (-\infty,-1) \cup \left(-1,\frac{3}{2}\right]$

47. $x \in \left(-\sqrt{5},\sqrt{5}\right)$

49. $x \in (-2,3)$

51. $x \in (-\infty,-1] \cup [3,4)$

53. $x \in (-\infty,-5) \cup (-5,-2) \cup (4,\infty)$

55. $\mathbb{R}$

57. $x = \frac{1}{2}$

59. $x = 0$

61. $t \in (0,3)$

63. over 145 units