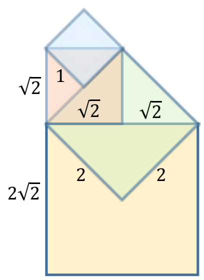
RD.4 Operations on Radical Expressions; Rationalization of Denominators



Unlike operations on fractions or decimals, sums and differences of many radicals cannot be simplified. For instance, we cannot combine $\sqrt{2}$ and $\sqrt{3}$, nor simplify expressions such as $\sqrt[3]{2} - 1$. These types of radical expressions can only be approximated with the aid of a calculator.

However, some radical expressions can be combined (added or subtracted) and simplified. For example, the sum of $2\sqrt{2}$ and $\sqrt{2}$ is $3\sqrt{2}$, similarly as $2x + x = 3x$.

In this section, first, we discuss the addition and subtraction of radical expressions. Then, we show how to work with radical expressions involving a combination of the four basic operations. Finally, we examine how to rationalize denominators of radical expressions.

Addition and Subtraction of Radical Expressions

Recall that to perform addition or subtraction of two variable terms we need these terms to be **like**. This is because the addition and subtraction of terms are performed by factoring out the variable "like" part of the terms as a common factor. For example,

$$x^2 + 3x^2 = (1+3)x^2 = 4x^2$$

The same strategy works for addition and subtraction of the same types of radicals or **radical terms** (terms containing radicals).

Definition 4.1 ► Radical terms containing radicals with the <u>same index</u> and the <u>same radicands</u> are referred to as **like radicals** or **like radical terms**.
For example,

$\sqrt{5x}$ and $2\sqrt{5x}$ are **like** (the indexes and the radicands are the same)

while

$5\sqrt{2}$ and $2\sqrt{5}$ are **<u>not</u> like** (the radicands are different)

and

$\sqrt{x}$ and $\sqrt[3]{x}$ are **<u>not</u> like radicals** (the indexes are different).

To **add** or **subtract like radical expressions** we **factor out the common radical** and any other common factor, if applicable. For example,

$$4\sqrt{2} + 3\sqrt{2} = (4+3)\sqrt{2} = 7\sqrt{2},$$

and

$$4xy\sqrt{2} - 3x\sqrt{2} = (4y+3)x\sqrt{2}.$$

Caution! Unlike radical expressions cannot be combined. For example, we are unable to perform the addition $\sqrt{6} + \sqrt{3}$. Such a sum can only be approximated using a calculator.

Notice that unlike radicals may become like if we simplify them first. For example, $\sqrt{200}$ and $\sqrt{50}$ are not like, but $\sqrt{200} = 10\sqrt{2}$ and $\sqrt{50} = 5\sqrt{2}$. Since $10\sqrt{2}$ and $5\sqrt{2}$ are like radical terms, they can be combined. So, we can perform, for example, the addition:

$$\sqrt{200} + \sqrt{50} = 10\sqrt{2} + 5\sqrt{2} = 15\sqrt{2}$$

Example 1 ► **Adding and Subtracting Radical Expressions**

Perform operations and simplify, if possible. Assume that all variables represent positive real numbers.

a. $5\sqrt{3}-8\sqrt{3}$

b. $3\sqrt[5]{2}-7x\sqrt[5]{2}+6\sqrt[5]{2}$

c. $7\sqrt{45}+\sqrt{80}-\sqrt{12}$

d. $3\sqrt[3]{y^5}-7\sqrt[3]{y^2}+\sqrt[3]{8y^8}$

e. $\sqrt{\frac{x}{16}}+2\sqrt{\frac{x^3}{9}}$

f. $\sqrt{25x-25}-\sqrt{9x-9}$

Solution ► **a.** To subtract like radicals, we combine their coefficients via factoring.

$$5\sqrt{3}-8\sqrt{3}=(5-8)\sqrt{3}=\mathbf{-3\sqrt{3}}$$

b. $3\sqrt[5]{2}-7x\sqrt[5]{2}+6\sqrt[5]{2}=(3-7x+6)\sqrt[5]{2}=(9-7x)\sqrt[5]{2}$

The brackets are essential here.

Note: Even if not all coefficients are like, factoring the common radical is a useful strategy that allows us to combine like radical expressions.

c. The expression $7\sqrt{45}+\sqrt{80}-\sqrt{12}$ consists of unlike radical terms, so they cannot be combined in this form. However, if we simplify the radicals, some of them may become like and then become possible to combine.

$$7\sqrt{45}+\sqrt{80}-\sqrt{12}=7\sqrt{9\cdot 5}+\sqrt{16\cdot 5}-\sqrt{4\cdot 3}=7\cdot 3\sqrt{5}+4\sqrt{5}-2\sqrt{3}$$

$$=21\sqrt{5}+4\sqrt{5}-2\sqrt{3}=\mathbf{25\sqrt{5}-2\sqrt{3}}$$

d. As in the previous example, we simplify each radical expression before attempting to combine them.

$$3\sqrt[3]{y^5}-5y\sqrt[3]{y^2}+\sqrt[5]{32y^7}=3y\sqrt[3]{y^2}-5y\sqrt[3]{y^2}+2y\sqrt[5]{y^2}$$

$$=(3y-5y)\sqrt[3]{y^2}+2y\sqrt[5]{y^2}=\mathbf{-2y\sqrt[3]{y^2}+2y\sqrt[5]{y^2}}$$

Remember to write the index with each radical.

Note: The last two radical expressions cannot be combined because of different indexes.

e. To perform the addition $\sqrt{\frac{x}{16}}+2\sqrt{\frac{x^3}{9}}$, we may simplify each radical expression first. Then, we add the expressions by bringing them to the least common denominator and finally, factor the common radical, as shown below.

$$\sqrt{\frac{x}{16}}+2\sqrt{\frac{x^3}{9}}=\frac{\sqrt{x}}{\sqrt{16}}+2\frac{\sqrt{x^3}}{\sqrt{9}}=\frac{\sqrt{x}}{4}+2\frac{\sqrt{x^3}}{3}=\frac{3\sqrt{x}+2\cdot 4\cdot x\sqrt{x}}{12}=\left(\mathbf{\frac{3+8x}{12}}\right)\boldsymbol{\sqrt{x}}$$

f. In an attempt to simplify radicals in the expression $\sqrt{25x^2-25}-\sqrt{9x^2-9}$, we factor each radicand first. So, we obtain

$$\sqrt{25x^2-25}-\sqrt{9x^2-9} = \sqrt{25(x^2-1)}-\sqrt{9(x^2-1)} = 5\sqrt{x^2-1}-3\sqrt{x^2-1}$$

$$= \mathbf{2\sqrt{x^2-1}}$$

> ***Caution!*** **The root of a sum <u>does not</u> equal the sum of the roots.** For example,
>
> $$\sqrt{5} = \sqrt{\mathbf{1+4}} \neq \sqrt{\mathbf{1}} + \sqrt{\mathbf{4}} = 1 + 2 = 3$$
>
> So, radicals such as $\sqrt{25x^2-25}$ or $\sqrt{9x^2-9}$ can be simplified only via factoring a perfect square out of their radicals while $\sqrt{x^2-1}$ cannot be simplified any further.

Multiplication of Radical Expressions with More than One Term

Similarly as in the case of multiplication of polynomials, multiplication of radical expressions where at least one factor consists of more than one term is performed by applying the distributive property.

Example 2 ► **Multiplying Radical Expressions with More than One Term**

Multiply and then simplify each product. Assume that all variables represent positive real numbers.

a. $5\sqrt{2}\left(3\sqrt{2x}-\sqrt{6}\right)$

b. $\sqrt[3]{x}\left(\sqrt[3]{3x^2}-\sqrt[3]{81x^2}\right)$

c. $\left(2\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-3\sqrt{2}\right)$

d. $\left(x\sqrt{x}-\sqrt{y}\right)\left(x\sqrt{x}+\sqrt{y}\right)$

e. $\left(3\sqrt{2}+2\sqrt[3]{x}\right)\left(3\sqrt{2}-2\sqrt[3]{x}\right)$

f. $\left(\sqrt{5y}+y\sqrt{y}\right)^2$

Solution ► **a.**

$$5\sqrt{2}\left(3\sqrt{2x}-\sqrt{6}\right) = 15\sqrt{4x} - 5\sqrt{2\cdot 2\cdot 3} = 15\cdot 2\sqrt{x} - 5\cdot 2\sqrt{3} = \mathbf{30\sqrt{x} - 10\sqrt{3}}$$

These are unlike terms. So, they cannot be combined.

$5\sqrt{2}\cdot 3\sqrt{2x} = 5\cdot 3\sqrt{2\cdot 2x}$

b.

simplification

$$\sqrt[3]{x}\left(\sqrt[3]{3x^2}-\sqrt[3]{81x^2}\right) = \sqrt[3]{3x^2\cdot x} - \sqrt[3]{81x^2\cdot x} = x\sqrt[3]{3} - 3x\sqrt[3]{3} = \mathbf{-2x\sqrt[3]{3}}$$

distribution

combining like terms

c. To multiply two binomial expressions involving radicals we may use the **FOIL** method. Recall that the acronym **FOIL** refers to multiplying the **F**irst, **O**uter, **I**nner, and **L**ast terms of the binomials.

$$\left(2\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-3\sqrt{2}\right)=\overset{\mathbf{F}}{2\cdot 3}-\overset{\mathbf{O}}{6\sqrt{3\cdot 2}}+\overset{\mathbf{I}}{\sqrt{2\cdot 3}}-\overset{\mathbf{L}}{3\cdot 2}=\cancel{6}-6\sqrt{6}+\sqrt{6}-\cancel{6}$$

$$=\mathbf{-5\sqrt{6}}$$

d. To multiply two conjugate binomial expressions we follow the difference of squares formula, $(a-b)(a+b)=a^2-b^2$. So, we obtain

$\left(\sqrt{x}\right)^2=x$

$$\left(x\sqrt{x}-\sqrt{y}\right)\left(x\sqrt{x}+\sqrt{y}\right)=\left(x\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2=x^2\cdot x-y=\boldsymbol{x^3-y}$$

square each factor

e. Similarly as in the previous example, we follow the difference of squares formula.

$$\left(3\sqrt{2}+2\sqrt[3]{x}\right)\left(3\sqrt{2}-2\sqrt[3]{x}\right)=\left(3\sqrt{2}\right)^2-\left(2\sqrt[3]{x}\right)^2=9\cdot 2-4\sqrt[3]{x^2}=\boldsymbol{18-4\sqrt[3]{x^2}}$$

f. To multiply two identical binomial expressions we follow the perfect square formula, $(a+b)(a+b)=a^2+2ab+b^2$. So, we obtain

$$\left(\sqrt{5y}+y\sqrt{y}\right)^2=\left(\sqrt{5y}\right)^2+2\left(\sqrt{5y}\right)\left(y\sqrt{y}\right)+\left(y\sqrt{y}\right)^2=5y+2y\sqrt{5y^2}+y^2y$$

$$=\boldsymbol{5y+2\sqrt{5}y^2+y^3}$$

Rationalization of Denominators

As mentioned in *section RD3*, a process of simplifying radicals involves rationalization of any emerging denominators. Similarly, a radical expression is not in its simplest form unless all its denominators are rational. This agreement originated before the days of calculators when computation was a tedious process performed by hand. Nevertheless, even in present time, the agreement of keeping denominators rational does not lose its validity, as we often work with variable radical expressions. For example, the expressions $\frac{2}{\sqrt{2}}$ and $\sqrt{2}$ are equivalent, as

$$\frac{2}{\sqrt{2}}=\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

Similarly, $\frac{x}{\sqrt{x}}$ is equivalent to $\sqrt{x}$, as

$$\frac{x}{\sqrt{x}}=\frac{x}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{x\sqrt{x}}{x}=\sqrt{x}$$

While one can argue that evaluating $\frac{2}{\sqrt{2}}$ is as easy as evaluating $\sqrt{2}$ when using a calculator, the expression $\sqrt{x}$ is definitely easier to use than $\frac{x}{\sqrt{x}}$ in any further algebraic manipulations.

Definition 4.2 ► The process of removing radicals from a denominator so that the denominator contains only rational numbers is called **rationalization** of the denominator.

Rationalization of denominators is carried out by multiplying the given fraction by a factor of 1, as shown in the next two examples.

Example 3 ► **Rationalizing Monomial Denominators**

Simplify, if possible. Leave the answer with a rational denominator. Assume that all variables represent positive real numbers.

a. $\frac{-1}{3\sqrt{5}}$ **b.** $\frac{5}{\sqrt[3]{32x}}$ **c.** $\sqrt[4]{\frac{81x^5}{y}}$

Solution ►

a. Notice that $\sqrt{5}$ can be converted to a rational number by multiplying it by another $\sqrt{5}$. Since the denominator of a fraction cannot be changed without changing the numerator in the same way, we multiply both, the numerator and denominator of $\frac{-1}{3\sqrt{5}}$ by $\sqrt{5}$. So, we obtain

$$\frac{-1}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-\sqrt{5}}{3 \cdot 5} = -\frac{\sqrt{5}}{15}$$

b. First, we may want to simplify the radical in the denominator. So, we have

$$\frac{5}{\sqrt[3]{32x}} = \frac{5}{\sqrt[3]{8 \cdot 4x}} = \frac{5}{2\sqrt[3]{4x}}$$

Then, notice that since $\sqrt[3]{4x} = \sqrt[3]{2^2x}$, it is enough to multiply it by $\sqrt[3]{2x^2}$ to nihilate the radical. This is because $\sqrt[3]{2^2x} \cdot \sqrt[3]{2x^2} = \sqrt[3]{2^3x^3} = 2x$. So, we proceed

$$\frac{5}{\sqrt[3]{32x}} = \frac{5}{2\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \frac{5\sqrt[3]{2x^2}}{2 \cdot 2x} = \frac{\mathbf{5\sqrt[3]{2x^2}}}{\mathbf{4x}}$$

Caution: A common mistake in the rationalization of $\sqrt[3]{4x}$ is the attempt to multiply it by a copy of $\sqrt[3]{4x}$. However, $\sqrt[3]{4x} \cdot \sqrt[3]{4x} = \sqrt[3]{16x^2} = 2\sqrt[3]{3x^2}$ is still not rational. This is because we work with a cubic root, not a square root. So, to rationalize $\sqrt[3]{4x}$ we must look for 'filling' the radicand to a perfect cube. This is achieved by multiplying $4x$ by $2x^2$ to get $8x^3$.

c. To simplify $\sqrt[4]{\frac{81x^5}{y}}$, first, we apply the quotient rule for radicals, then simplify the radical in the numerator, and finally, rationalize the denominator. So, we have

$$\sqrt[4]{\frac{81x^5}{y}} = \frac{\sqrt[4]{81x^5}}{\sqrt[4]{y}} = \frac{3x\sqrt[4]{x}}{\sqrt[4]{y}} \cdot \frac{\sqrt[4]{y^3}}{\sqrt[4]{y^3}} = \frac{\mathbf{3x\sqrt[4]{xy^3}}}{\mathbf{y}}$$

To rationalize a binomial containing square roots, such as $2-\sqrt{x}$ or $\sqrt{2}-\sqrt{3}$, we need to find a way to square each term separately. This can be achieved through multiplying by a conjugate binomial, in order to benefit from the difference of squares formula. In particular, we can rationalize denominators in expressions below as follows:

$$\frac{1}{2-\sqrt{x}}=\frac{1}{(2-\sqrt{x})}\cdot\frac{(2+\sqrt{x})}{(2+\sqrt{x})}=\frac{\mathbf{2}+\sqrt{\boldsymbol{x}}}{\mathbf{4}-\boldsymbol{x}}$$

Apply the difference of squares formula: $(a-b)(a+b)=a^2-b^2$

or

$$\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}=\frac{\sqrt{2}}{(\sqrt{2}+\sqrt{3})}\cdot\frac{(\sqrt{2}-\sqrt{3})}{(\sqrt{2}-\sqrt{3})}=\frac{2-\sqrt{6}}{2-3}=\frac{2-\sqrt{6}}{-1}=\sqrt{\mathbf{6}}-\mathbf{2}$$

Example 4 ► **Rationalizing Binomial Denominators**

Rationalize each denominator and simplify, if possible. Assume that all variables represent positive real numbers.

a. $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ **b.** $\frac{\sqrt{xy}}{2\sqrt{x}-\sqrt{y}}$

Solution ►

a. $\frac{1-\sqrt{3}}{1+\sqrt{3}}\cdot\frac{(1-\sqrt{3})}{(1-\sqrt{3})}=\frac{1-2\sqrt{3}+3}{1-3}=\frac{4-2\sqrt{3}}{-2}\overset{\text{factor}}{=}\frac{\cancel{-2}(-2+\sqrt{3})}{\cancel{-2}}=\sqrt{\mathbf{3}}-\mathbf{2}$

b. $\frac{\sqrt{xy}}{2\sqrt{x}-\sqrt{y}}\cdot\frac{(2\sqrt{x}+\sqrt{y})}{(2\sqrt{x}+\sqrt{y})}=\frac{2x\sqrt{y}+y\sqrt{x}}{4x-y}$

Some of the challenges in algebraic manipulations involve simplifying quotients with radical expressions, such as $\frac{4-2\sqrt{3}}{-2}$, which appeared in the solution to *Example 4a*. The key concept that allows us to simplify such expressions is **factoring**, as only common factors can be reduced.

Example 5 ► **Writing Quotients with Radicals in Lowest Terms**

Write each quotient in lowest terms.

a. $\frac{15-6\sqrt{5}}{6}$ **b.** $\frac{3x+\sqrt{8x^2}}{9x}$

Solution ►

a. To reduce this quotient to the lowest terms we may factor the numerator first,

$$\frac{15-6\sqrt{5}}{6}=\frac{\cancel{3}(5-2\sqrt{5})}{\cancel{6}_{2}}=\frac{\mathbf{5}-\mathbf{2}\sqrt{\mathbf{5}}}{\mathbf{2}},$$

or alternatively, rewrite the quotient into two fractions and then simplify,

$$\frac{15-6\sqrt{5}}{6}=\frac{15}{6}-\frac{6\sqrt{5}}{6}=\frac{\mathbf{5}}{\mathbf{2}}-\sqrt{\mathbf{5}}.$$

Caution: Here are the common errors to avoid:

$\cancel{\frac{15-\cancel{6}\sqrt{5}}{\cancel{6}}=15-\sqrt{5}}$ - only common factors can be reduced!

$\cancel{\frac{15-6\sqrt{5}}{6}=\frac{\overset{3}{\cancel{9}}\sqrt{5}}{\cancel{6}}}=\frac{3\sqrt{5}}{2}$ - subtraction is performed after multiplication!

b. To reduce this quotient to the lowest terms, we simplify the radical and factor the numerator first. So,

$$\frac{3x+\sqrt{8x^2}}{6x}=\frac{3x+2x\sqrt{2}}{6x}=\frac{\cancel{x}(3+2\sqrt{2})}{6\cancel{x}}=\frac{\mathbf{3+2\sqrt{2}}}{\mathbf{6}}$$

This expression cannot be simplified any further.

RD.4 Exercises

Vocabulary Check *Complete each blank with the most appropriate term or phrase from the given list:* ***coefficients, denominator, distributive, factoring, like, rationalize, simplified.***

1. Radicals that have the same index and the same radicand are called __________ radicals.

2. Some unlike radicals can become like radicals after they are _____________.

3. Like radicals can be added by combining their _______________ and then multiplying this sum by the common radical.

4. To multiply radical expressions containing more than one term, we use the _____________ property.

5. To _____________ the denominator means to rewrite an expression that has a radical in its denominator as an equivalent expression that does not have one.

6. To rationalize a denominator with two terms involving square roots, multiply both the numerator and the denominator by the conjugate of the _____________.

7. The key strategy for simplifying quotients such as $\frac{a+b\sqrt{c}}{d}$ is _____________ the numerator, if possible.

Concept Check

8. A student claims that $24 - 4\sqrt{x} = 20\sqrt{x}$ because for $x = 1$ both sides of the equation equal to 20. Is this a valid justification? Explain.

9. For most values of a and b, $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$. Are there any real numbers a and b such that $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$? Justify your answer.

Perform operations and simplify, if possible. Assume that all variables represent positive real numbers.

10. $2\sqrt{3} + 5\sqrt{3}$

11. $6\sqrt[3]{x} - 4\sqrt[3]{x}$

12. $9y\sqrt{3x} + 4y\sqrt{3x}$

13. $12a\sqrt{5b} - 4a\sqrt{5b}$

14. $5\sqrt{32} - 3\sqrt{8} + 2\sqrt{3}$

15. $-2\sqrt{48} + 4\sqrt{75} - \sqrt{5}$

16. $\sqrt[3]{16} + 3\sqrt[3]{54}$

17. $\sqrt[4]{32} - 3\sqrt[4]{2}$

18. $\sqrt{5a} + 2\sqrt{45a^3}$

19. $\sqrt[3]{24x} - \sqrt[3]{3x^4}$

20. $4\sqrt{x^3} - 2\sqrt{9x}$

21. $7\sqrt{27x^3} + \sqrt{3x}$

22. $6\sqrt{18x} - \sqrt{32x} + 2\sqrt{50x}$

23. $2\sqrt{128a} - \sqrt{98a} + 2\sqrt{72a}$

24. $\sqrt[3]{6x^4} + \sqrt[3]{48x} - \sqrt[3]{6x}$

25. $9\sqrt{27y^2} - 14\sqrt{108y^2} + 2\sqrt{48y^2}$

26. $3\sqrt{98n^2} - 5\sqrt{32n^2} - 3\sqrt{18n^2}$

27. $-4y\sqrt{xy^3} + 7x\sqrt{x^3y}$

28. $6a\sqrt{ab^5} - 9b\sqrt{a^3b}$

29. $\sqrt[3]{-125p^9} + p\sqrt[3]{-8p^6}$

30. $3\sqrt[4]{x^5y} + 2x\sqrt[4]{xy}$

31. $\sqrt{125a^5} - 2\sqrt[3]{125a^4}$

32. $x\sqrt[3]{16x} + \sqrt{2} - \sqrt[3]{2x^4}$

33. $\sqrt{9a-9} + \sqrt{a-1}$

34. $\sqrt{4x+12} - \sqrt{x+3}$

35. $\sqrt{x^3 - x^2} - \sqrt{4x-4}$

36. $\sqrt{25x-25} - \sqrt{x^3 - x^2}$

37. $\frac{4\sqrt{3}}{3} - \frac{2\sqrt{3}}{9}$

38. $\frac{\sqrt{27}}{2} - \frac{3\sqrt{3}}{4}$

39. $\sqrt{\frac{49}{x^4}} + \sqrt{\frac{81}{x^8}}$

40. $2a\sqrt[4]{\frac{a}{16}} - 5a\sqrt[4]{\frac{a}{81}}$

41. $-4\sqrt[3]{\frac{4}{y^9}} + 3\sqrt[3]{\frac{9}{y^{12}}}$

Discussion Point

42. A student simplifies an expression *incorrectly*:

$$\begin{aligned}\sqrt{8} + \sqrt[3]{16} &\overset{?}{=} \sqrt{4 \cdot 2} + \sqrt[3]{8 \cdot 2} \\ &\overset{?}{=} \sqrt{4} \cdot \sqrt{2} + \sqrt[3]{8} \cdot \sqrt[3]{2} \\ &\overset{?}{=} 2\sqrt{2} + 2\sqrt[3]{2} \\ &\overset{?}{=} 4\sqrt{4} \\ &\overset{?}{=} 8\end{aligned}$$

Explain any errors that the student made. What would you do differently?

Concept Check

43. Match each expression from ***Column I*** with the equivalent expression in ***Column II***. Assume that A and B represent positive real numbers.

Column I	***Column II***
A. $(A+\sqrt{B})(A-\sqrt{B})$	**a.** $A-B$
B. $(\sqrt{A}+B)(\sqrt{A}-B)$	**b.** $A+2B\sqrt{A}+B^2$
C. $(\sqrt{A}+\sqrt{B})(\sqrt{A}-\sqrt{B})$	**c.** $A-B^2$
D. $(\sqrt{A}+\sqrt{B})^2$	**d.** $A-2\sqrt{AB}+B$
E. $(\sqrt{A}-\sqrt{B})^2$	**e.** A^2-B
F. $(\sqrt{A}+B)^2$	**f.** $A+2\sqrt{AB}+B$

Multiply, and then simplify each product. Assume that all variables represent positive real numbers.

44. $\sqrt{5}(3-2\sqrt{5})$

45. $\sqrt{3}(3\sqrt{3}-\sqrt{2})$

46. $\sqrt{2}(5\sqrt{2}-\sqrt{10})$

47. $\sqrt{3}(-4\sqrt{3}+\sqrt{6})$

48. $\sqrt[3]{2}(\sqrt[3]{4}-2\sqrt[3]{32})$

49. $\sqrt[3]{3}(\sqrt[3]{9}+2\sqrt[3]{21})$

50. $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

51. $(\sqrt{5}+\sqrt{7})(\sqrt{5}-\sqrt{7})$

52. $(2\sqrt{3}+5)(2\sqrt{3}-5)$

53. $(6+3\sqrt{2})(6-3\sqrt{2})$

54. $(5-\sqrt{5})^2$

55. $(\sqrt{2}+3)^2$

56. $(\sqrt{a}+5\sqrt{b})(\sqrt{a}-5\sqrt{b})$

57. $(2\sqrt{x}-3\sqrt{y})(2\sqrt{x}+3\sqrt{y})$

58. $(\sqrt{3}+\sqrt{6})^2$

59. $(\sqrt{5}-\sqrt{10})^2$

60. $(2\sqrt{5}+3\sqrt{2})^2$

61. $(2\sqrt{3}-5\sqrt{2})^2$

62. $(4\sqrt{3}-5)(\sqrt{3}-2)$

63. $(4\sqrt{5}+3\sqrt{3})(3\sqrt{5}-2\sqrt{3})$

64. $(\sqrt[3]{2y}-5)(\sqrt[3]{2y}+1)$

65. $(\sqrt{x+5}-3)(\sqrt{x+5}+3)$

66. $(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})$

67. $(\sqrt{x+2}+\sqrt{x-2})^2$

Concept Check

Given $f(x)$ and $g(x)$, find $(f+g)(x)$ and $(fg)(x)$.

68. $f(x)=5x\sqrt{20x}$ and $g(x)=3\sqrt{5x^3}$

69. $f(x)=2x\sqrt[4]{64x}$ and $g(x)=-3\sqrt[4]{4x^5}$

Rationalize each denominator and simplify, if possible. Assume that all variables represent positive real numbers.

70. $\frac{\sqrt{5}}{2\sqrt{2}}$

71. $\frac{3}{5\sqrt{3}}$

72. $\frac{12}{\sqrt{6}}$

73. $-\frac{15}{\sqrt{24}}$

74. $-\frac{10}{\sqrt{20}}$

75. $\sqrt{\frac{3x}{20}}$

76. $\sqrt{\frac{5y}{32}}$

77. $\frac{\sqrt[3]{7a}}{\sqrt[3]{3b}}$

78. $\frac{\sqrt[3]{2y^4}}{\sqrt[3]{6x^4}}$

79. $\frac{\sqrt[3]{3n^4}}{\sqrt[3]{5m^2}}$

80. $\frac{pq}{\sqrt[4]{p^3q}}$

81. $\frac{2x}{\sqrt[5]{18x^8}}$

82. $\frac{17}{6+\sqrt{2}}$

83. $\frac{4}{3-\sqrt{5}}$

84. $\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

85. $\frac{6\sqrt{3}}{3\sqrt{2}-\sqrt{3}}$

86. $\frac{3}{3\sqrt{5}+2\sqrt{3}}$

87. $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}+5\sqrt{2}}$

88. $\frac{m-4}{\sqrt{m}+2}$

89. $\frac{4}{\sqrt{x}-2\sqrt{y}}$

90. $\frac{\sqrt{3}+2\sqrt{x}}{\sqrt{3}-2\sqrt{x}}$

91. $\frac{\sqrt{x}-2}{3\sqrt{x}+\sqrt{y}}$

92. $\frac{2\sqrt{a}}{\sqrt{a}-\sqrt{b}}$

93. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Write each quotient in lowest terms. Assume that all variables represent positive real numbers.

94. $\frac{10-20\sqrt{5}}{10}$

95. $\frac{12+6\sqrt{3}}{6}$

96. $\frac{12-9\sqrt{72}}{18}$

97. $\frac{2x+\sqrt{8x^2}}{2x}$

98. $\frac{6p-\sqrt{24p^3}}{3p}$

99. $\frac{9x+\sqrt{18}}{15}$

Discussion Point

100. In a certain problem in trigonometry, a student obtained the answer $\frac{\sqrt{3}+1}{1-\sqrt{3}}$. The textbook answer to this problem was $-2-\sqrt{3}$. Was the student's answer equivalent to the textbook answer?

Analytic Skills

Solve each problem.

101. The Great Pyramid at Giza has a square base with an area of 52,900 m^2. What is the perimeter of its base?

102. The areas of two types of square floor tiles sold at a home improvement store are shown. How much longer is the side of the larger tile? Express the answer as a radical in simplest form and as a decimal to the nearest tenth.

Area = $72\ cm^2$

Area = $128\ cm^2$